The Problem of Consistency in the Design of Fitt's Law Experiments: Consider either Target Distance and Width or Movement Form and Scale

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ABSTRACT

An intriguing anomaly of the usual way of designing Fitts' law experiments in experimental psychology and HCI is exposed: experiments are traditionally designed so as to carefully balance two ancillary factors, target distance *D* and target width *W*, but not task difficulty, the factor unanimously thought to be crucial. Troubling factor confounds and hence quantitative estimation errors result from this inconsistency. The problem, it is suggested, may be fixed if the equivocalness of the fractional expression *D/W* that appears on the right-hand side of Fitts' law equations is acknowledged. This two-degree-of-freedom expression can be taken to specify either *D* and *W* or the form *F* and the scale *S* of the movement task. The paper ends up with practical recommendations for the design of consistent Fitts' law experiments. In most cases eliminating one factor will allow a safer estimation of Fitts' law parameters, while simplifying the experimental work.

Author Keywords

Fitts' law, experimental design, method, theory.

ACM Classification Keywords

H.5.2. User Interfaces: Evaluation/methodology, theory and method.

INTRODUCTION: FITTS' LAW, A VALUABLE CONCEPTUAL TOOL FOR HCI RESEARCH

Fitts' law is an empirical relation which states that the mean time μ_T it takes people to reach an object of width *W* located at a distance *D* is lawfully dependent on the ratio *D/W*. In the particular formulation that is widely received in the HCI community [17], the law reads

$$\mu_T = k_1 + k_2 * \log_2(D/W+1), \qquad (1)$$

where the term $\log_2(D/W+1)$ is known as the task's index of difficulty (*ID*) and k_1 and k_2 stand for empirically adjustable coefficients. First described by Fitts [4,5], the law has been routinely exploited in HCI since the very outset of the graphical user interface [2]. It is useful in HCI because it makes it possible to both predict human pointing performance theoretically (e.g., to compute optimal layouts of soft keyboards [18,32]) and evaluate interfaces in practice [17]. Thanks to Eq. 1 one can quantitatively characterize with just two numbers, k_1 and k_2, the pointing performance attainable by users of any specific input device, target layout or interaction technique. Pointing being a central building block of interaction in graphical interfaces, the application scope of the law in HCI is very large [11]. Thanks to Fitts' law, a great deal of experimental effort can be saved in the context of interface evaluation. Eq. 1 is reliable enough that in principle measuring μ_T at just two well-contrasted levels of the *ID* (like 2 vs. 8 bits)[1] would suffice to obtain suitable estimates of coefficients k_1 and k_2 for each of the devices, layouts, or techniques under consideration.

Since Fitts [4,] the specialized psychological literature has produced a number of competing mathematical formulations of Fitts' law involving a logarithmic [28], a power [21], or a linear [23] relation. Advocating Fitts' information-theoretic interpretation of Fitts' law, MacKenzie [17] produced solid arguments, based on Shannon's Theorem 17 [24], in favor of the specific logarithmic formulation shown in Eq. 1. Note, however, that Fitts' law equations are all of the form

$$\mu_T = \mathrm{f}\,(D/W), \qquad (2)$$

where f means "is a monotonically increasing function of". This formulation of Fitts' law deliberately ignores all the controversial detail, however interesting and important,[2] and only retains what all theorists seem to have agreed on since Fitts' seminal work. For our present purposes, this generic formulation will reveal quite useful.

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[1] Linear interpolation being generally quite safe within such a range of *ID* (typically r^2>.9) [26], little extra information can be obtained by measuring μ_T at intermediate levels.

[2] A dozen variants of Fitts' law equation, all belonging to the equivalence class of Equation 2, are listed by Plamondon and Alimi [22].

AN ANOMALY IN THE STANDARD WAY OF DESIGNING FITTS' LAW EXPERIMENTS

The Usual Way of Designing Fitts' Law Experiments

In HCI as well as in experimental psychology it has been customary since Fitts [4] to resort to experimental designs that cross factors *D* and *W* orthogonally so as to obtain a suitable range of ratios and hence of *ID*s.

Table 1 presents the 16 conditions of one of Fitts' [4] experiments.[3] Fitts' design crossed factors *D* and *W* orthogonally (let us call this a *D***W* design), meaning that for each of the *m* values of variable *D* to be used in the experiment, all the *n* values of variable *W* believed to be of interest were considered, and so the experimental design included *mn* cells or conditions. With that *D***W* design the distribution of *W* was the same for all values of *D* (on average, *W* was a constant 0.234 in.) and conversely the distribution of *D* was the same for all values of *W* (mean *D* was a constant 15.0 in.). This property of total mutual independence is reflected by the fact that the spread of data points in the scatter plot of *D* vs. *W* is rectangular (Fig. 1, top).

D (in.)	**4**	**4**	**4**	**4**	**8**	**8**	**8**	**8**	**16**	**16**	**16**	**16**	**32**	**32**	**32**	**32**
W (in.)	**1/2**	**1/4**	**1/8**	**1/16**	**1/2**	**1/4**	**1/8**	**1/16**	**1/2**	**1/4**	**1/8**	**1/16**	**1/2**	**1/4**	**1/8**	**1/16**
D/*W* (-)	8	16	32	64	16	32	64	128	32	64	128	256	64	128	256	512
ID (bit)	4.0	5.0	6.0	7.0	5.0	6.0	7.0	8.0	6.0	7.0	8.0	9.0	7.0	8.0	9.0	10.0

Table 1. The design of Fitts' [4] disc-transfer experiment (tabulated here is Fitts' own version of the *ID* = $\log_2(2D/W)$).

However, the suitability of the *D***W* design in a Fitts' law study is questionable. Factors *D* and *W* have just technical relevance, these factors representing just the handles we need to control the ratio *D/W*, the sole determinant of the all-important *ID*. Realization that the *dimensionless* ratio *D/W* — rather than *lengths D* and *W* — is indeed the crucial variable for predicting movement time in a Fitts task may be viewed as Fitts' [4] main insight, an insight that has been supported by a considerable body of empirical evidence in fifty years of subsequent research [22]. But if the ratio *D/W* is the main causal factor, then why did Fitts have recourse to a *D***W* design? The trouble, as we will see in the next sub-section, is that such a design does not guarantee the experimental *isolation* of the effect of the ratio *D/W*.

Ever since Fitts' pioneering work half a century ago the *D***W* design has been an unchallenged methodological norm for Fitts' law experimentation in HCI, ergonomics, as well as basic psychology, the incidence of departures from this design norm in the whole literature on Fitts' law being presumably well below 1%. We will focus on Fitts' seminal work, but any less famous example could have been used to make the point. At issue is the validity of the established way of designing Fitts' law experiments.

[3] That particular experiment of Fitts used exactly the same design as his famous stylus-tapping experiment. We focus on it below because, as we will see, it failed in quite an instructive way.

Factor Confounds in Fitts' Design

While Fitts' experimental design (Table 1) carefully balances the variation of variables *D* and *W* relative to each other, it fails to isolate the variation of the crucial ratio *D/W*. Because the two length measures *D* and *W* are made to vary concomitantly with the *ID*, we have two factor confounds.

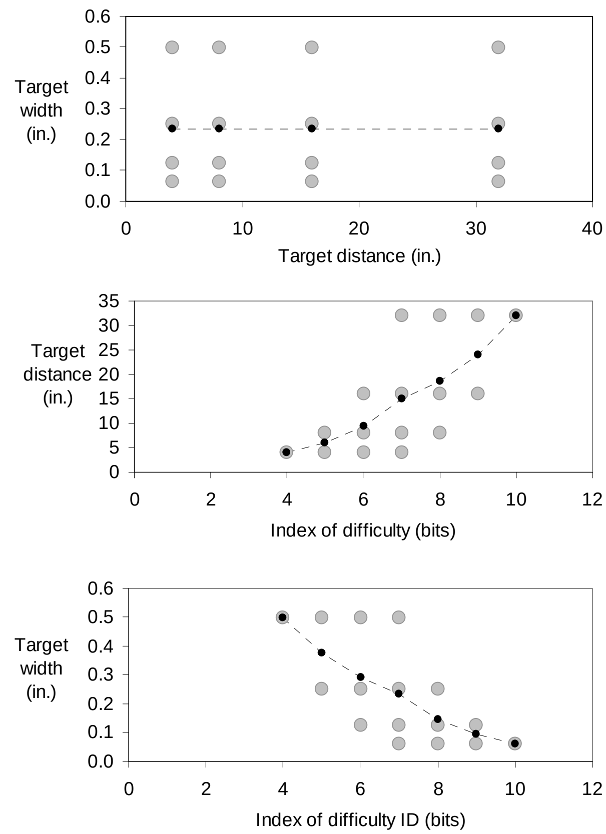

Figure 1. Co-variation patterns among independent variables in Fitts' [4] design for his disc-transfer experiment. The data points colored in light-gray represent the 16 experimental conditions of the study. Small black circles represent mean values of *y* for each value of *x*.

Consider the middle graph of Fig. 1. Fitts manipulated the *ID* from 4 to 10 bits (i.e., over a 1-2.5 range) but in his design the *ID* manipulation caused mean target distance to inflate concomitantly from 10.16cm to 81.28cm (from 4 to 32in.), which represents no less than an eight-fold variation. The dependence is strong and systematic: on average target distance is raised by 11.7cm for each extra bit of information. Therefore, upon finding that μ_T is lengthened as the *ID* is raised, how will one be sure that the effect of the *ID* (in bits), estimated on average, has not been contaminated by the effect of moving the target farther away (in cm)?

There is a similar confound between the *ID* and factor *W* (Fig. 1, bottom). Raising the *ID* from 4 to 10bits in Fitts' design means concomitantly reducing the mean value of *W* from 1/2in. (1.28cm) to 1/16in. (0.16cm), again an eight-fold variation. Each extra bit of information going along with a 1.8mm reduction of target size, how will one be sure that the mean effect of the *ID* (in bits) has not been contaminated by the concomitant variation of *W* (in cm)?

Are *D* and *W* Theoretically Relevant Factors? Conceptual Irresolution in the Basic Literature

One possible response to the above concern would be that the two confounds are of immaterial importance because, as already suggested above, in Fitts' paradigm lengths *D* and *W* do not stand for theoretically important quantities. Such a view, however, would conflict with widespread assumptions. For example, in one of the most influential contributions to the literature, Meyer et al. [21] introduce the results of an analysis of variance in these terms:

> "The purpose of this analysis was to test whether target distance and width had proportionally compensatory effects, as implied by Fitts' law. If such compensation holds, then *T* should vary directly with *D/W*, and neither *D* nor *W* should have any residual effect on *T* beyond their contributions to the effect of *D/W*. " (p. 354).

What is explicitly suggested here is that experimenters are facing *three* independent variables, not only the dimensionless ratio *D/W*, but also lengths *D* and *W*.[4] But if these three variables are all candidate causal factors, why then adopt the *D*W* design inherited from Fitts, which considers only two factors?[5] Assuming, like Meyer et al., that *D*, *W*, and *D/W* are all relevant quantities, the strong mutual dependences shown in Fig. 1, to be found in *any D*W* design, are indeed problematic.

MIS-ESTIMATING FITTS' LAW PARAMETERS: THE EXAMPLE OF FITTS' DATA

Failure to adequately balance a factor in an experimental design is risky because the effect of this factor may be inadvertently contaminated by other influences. In fact, a careful inspection of Fitts' 1954 data, collected in three experiments all with the same *D*W* design, reveals that Fitts was not evenly successful at isolating the *ID* effect on μ_T. Comparing the data of his disc-transfer experiment (Exp. 2) with those of his very well known stylus-tapping experiment (Exp. 1), one striking difference can be noticed.[6]

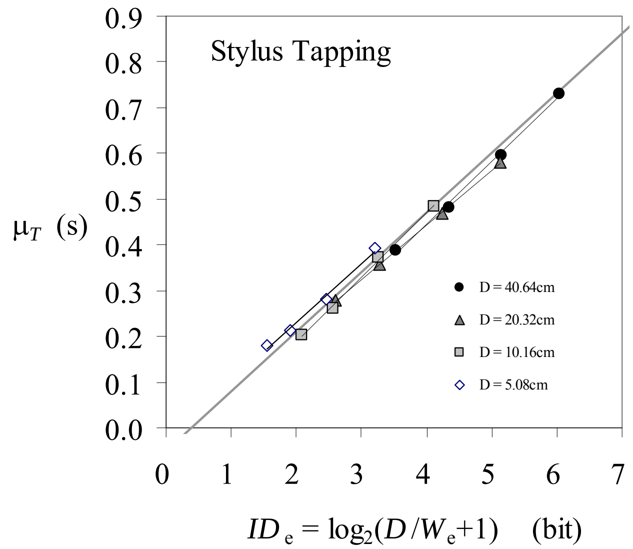

Figure 2. The data of Fitts' Exp. 1 on tapping (shown on the *x* axis is the effective index of difficulty).

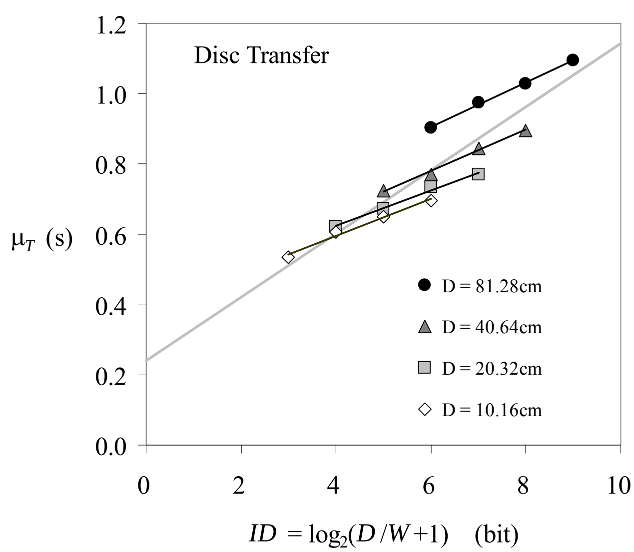

Figure 3. The data of Fitts' Exp. 2 on disc transfer (this task imposing a 0% error rate, the *x* axis shows nominal *ID*).

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[4] The view that Fitts' paradigm involves three causal variables has been expressed numerous times, notably in another widely cited paper by Plamondon and Alimi [22] (p. 280), who use much the same words as Meyer et al.

[5] Meyer et al. [21] used the usual *D*W* design in their two experiments. Thus they managed to obtain approximately zero correlation between *D* and *W*, but their design caused both *D* and *W* to co-vary dramatically with the *ID*. Computing from their Table 3 (p. 352), the equations of best fit are $D = 8.2ID + 0.2$ ($r^2 = .994$) and $W = -1.3ID + 6.9$ ($r^2=.943$). What should not be overlooked here is that these strong stochastic links concern supposedly *independent* variables.

[6] For reasons to be explained below, the variable *D* that appears as a parameter in Figs. 2 and 3 is best understood here as measuring the *scale* of the movement task.

Figs. 2 and 3 plot Fitts' law as usual, expressing μ_T as a function of the *ID*, but they use different graphical symbols to distinguish the different levels of *D*. Fig. 2 shows that in Fitts' famous tapping experiment all 16 data points were fairly well aligned, showing a massive effect of the *ID* and virtually no influence of *D*. The outcome was less satisfactory in his less famous disc-transfer experiment (Fig. 3), with four well-segregated alignments of data points, one for each of the four levels of *D*, suggesting a substantial effect of distance *D* beside the obvious effect of the *ID*. For example at *ID* = 6bits (the only level of *ID* where Fitts' design offered a measurement of μ_T for each of the four levels of *D*), μ_T varied from 697ms for the shortest target distance up to 902ms for the largest, no less than a 30% increase. Judging by the parallelism of the four regression lines, the effects of factors *D* and *ID* look essentially additive in this data set.

In fact the very presence of an effect of factor *D* in a *D***W*-designed Fitts' law experiment is liable to bias the estimation of the crucial slope of Fitts' law, whose inverse (in bits/s) measures information throughput [2,31]. The standard method of estimating that slope in both basic and applied research consists of fitting a linear equation indiscriminately to the whole pool of μ_T measures, thus ignoring factor *D*. For the data of Fig. 3 that technique yields the equation $\mu_T = 0.090ID + 0.5$ ($r^2 = .844$) illustrated as a gray line crossing the whole graph. But Fig. 3 also shows the four regression lines that obtain when the μ_T vs. *ID* regression is computed separately for each level of *D*. With this latter, more cautious method the estimates of Fitts' law slope are 53, 50, 59, and 64ms/bit for *D* = 10, 20, 40, and 80cm, respectively, and so the average slope over the experiment is 56ms/bit, rather than 90ms/bit.

Estimating Fitts' law slope separately is almost certainly preferable because it means estimating the effect of the *ID* at constant levels (i.e., independently) of *D*. Estimating the slope globally without taking the variation of *D* into account can only result in a more or less biased estimate, the effect of the *ID* being contaminated by that of *D*. Of course *D* may exert no detectable influence on μ_T, as was the case in Fitts' tapping experiment (Fig. 2). However, in less fortunate cases like Fitts' disc-transfer experiment, the pooling method introduced by Fitts will delivers biased estimates—what that method actually produces in Fig. 3 is no less than a 60% over-estimation of the slope.

A first conclusion is that a bias-free assessment of Fitts' law parameters can be obtained from data gathered with the conventional *D***W* design provided one computes the linear regression separately for each level of *D*, rather than globally over pooled *D* levels. However, one may not be content with this practical solution because correcting the shortcomings of one's method implies accepting these shortcomings in the first place. Below we try to understand precisely what is wrong in the logic underlying the standard way of designing Fitts' law experiments.

EXPLAINING THE ANOMALY: THE EQUIVOCALNESS OF FRACTIONAL EXPRESSION *D/W*

The methodological anomaly we want to understand manifests itself through two symptoms. The most obvious symptom is that the set of independent variables of the experimentation fails to coincide with the set of causal variables of the theory. It is a general rule of experimental science that investigators manipulate systematically, in carefully balanced designs, the variables that they assume, in light of their theory, to play a causal role. There has been unanimous agreement since Fitts that the *ID*, and ultimately the dimensionless ratio *D/W*, is the core determinant of μ_T in Fitts tasks. Yet the ratio in question never appears as an explicitly balanced factor in experimental designs, investigators instead manipulating *D* and *W*, two variables of little importance.

The other, related symptom is a persistent state of irresolution in the literature as to the theoretical status of the three measures that authors identify on the right-hand side of Eq. 2, length *D*, length *W*, and the dimensionless ratio *D/W*. Meyer et al.'s [21] above-quoted statement is one of many that express embarrassment on this issue: in addition to the *ID*, should not researchers try to *also* evaluate the effects of the two length measures that compose the dimensionless ratio?

I will suggest that this recurring question must be answered negatively. What must be realized is that the measures *D*, *W*, and *D/W* form a trio *with only two degrees of freedom* (DOF): the point, quite simply, is that it is impossible to vary one member of the trio independently of the other two. Thus the widely shared concern expressed in Meyer et al.'s above quote about the way in which these three factors combine their effects seems ill grounded. If there is no way to define these three measures independently of one another, it is logically unsound to inquire into three separate causal influences in a Fitts' law experiment. There is room for just two independent factors in the experimentation and, accordingly, for just two causal entities in the theory.

What are the Problem's Degrees of Freedom? A Simple Geometrical Analogy

The obvious question that arises next is, What are the two DOF of the trio? To clarify the matter, let us start with a simple geometrical analogy likening the two DOF of fractional expression *D/W* to the two DOF required to localize a point in a plane (Fig. 4) [9,10].

Point P can be localized in 2D space either by a pair of Cartesian coordinates or by a pair of polar coordinates. The former consist of the norms of the component vectors $\overrightarrow{Ox}$ and $\overrightarrow{Oy}$, the latter consist of the orientation ϕ and the norm of the resultant vector $\overrightarrow{OP}$. Thus point P can be uniquely localized either by means of two *length* measures (the width and height of rectangle OyPx of Fig. 4) or, alternatively, by means of one measure of shape or *form*, the angle ϕ, which reflects the rectangle's aspect ratio, supplemented by one measure of *scale*, conventionally the length of the rectangle's diagonal. Listing up the measures we have mentioned, we have the width and the height of the rectangle O*y*P*x* as well as its form and its scale. However, one is facing two alternative descriptions: two descriptors must be chosen among these four. One cannot reason about three quantities like, say, the form, the width, and the height of the rectangle because localizing a point in a plane involves only two DOF. Moreover, a hybridized Cartesian/polar pair like, say, the rectangle's height and form would be confusing because an independent manipulation of the latter factor requires that not simply the height but both the height and the width (i.e., the scale) of the rectangle be conserved.

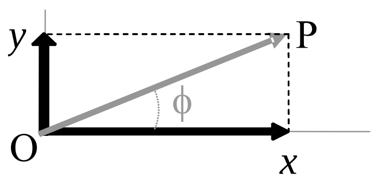

Figure 4. The Cartesian vs. polar specification of a point in planar geometry.

Returning to the fractional expression *D/W* of Fitts' law, let us now make the analogy explicit. The expression *D/W* of Eq. 2 can be viewed to exhibit
- *either* its numerator and its denominator, lengths *D* and *W*, which are the operands of a doable division,
- *or* the quotient[7] $Q_{D/W}$ of the completed division of *D* by *W* as well as some scale measure, say the magnitude of the numerator.

These two alternative understandings of expression *D/W* can be labeled, by analogy, as Cartesian and polar, respectively. These labels identify quite accurately the two unequivocal ways of understanding the expression *D/W* of Eq. 2. Most importantly, they help realize that these understandings are mutually exclusive because the expression *D/W* (and in fact Fitts' task) offer just two DOF. One may certainly investigate theoretically and/or experimentally the effects of *D* and *W* on μ_T, but such an inquiry demands that one forget about the possible effect of the quotient $Q_{D/W}$ — and hence of the *ID*. Alternatively, one may want to study the effects of the *ID* and scale, but then one must forget about such quantities as *D* and *W*.

This analysis may look rather formal to the reader, but it is important to keep in mind that a formalism like a Fitts' law equation is useful only to the extent that each of its algebraic symbols can be mapped onto some identified entity of the substantive theory [20] as well as onto some concrete measure of the laboratory [10]. At issue here is an inescapable choice between two mutually exclusive theoretical views of Fitts' law, which disagree about something quite crucial—the number and the identity of the independent variables represented on the right-hand side of Fitts' law equations. Eq. 2 may be read as $\mu_T = f(D, W)$, meaning that μ_T depends on both factors of the Cartesian description, which have the dimension of lengths. Alternatively, if the equation is taken in the polar sense to read $\mu_T = f(Q_{D/W})$, its meaning is quite different: μ_T depends on a *single* factor, a dimensionless quotient. Notice that the latter interpretation, which obviously corresponds to the established understanding of Fitts' law, implies that scale, one of the two factors of the polar description, exerts no effect. Indeed, it is an important, if seldom recognized, aspect of Fitts' law that μ_T, within limits, is fairly insensitive to task rescaling (isochrony).

The quotient $Q_{D/W}$ quantifies the *form* of a Fitts task in 1D space just as the quotient $Q_{x/y}$ quantifies the form of rectangle OyPx in 2D space (Fig. 4). As for task *scale*, we may think of several measures: the quotient being known, the scale of the fractional expression *D*/*W* of Eq. 2 is equally well specified by its numerator and its denominator. By default below, task scale will be measured by *D*, the variable that experimenters manipulate to control the average extent of the required movement.

In the rest of the paper the two possible understandings of Eq. 2 will be labeled as the Distance*Width (*D*W*) vs. the Form*Scale (*F*S*) understandings.

Woodworth's vs. Fitts' View

The *D*W* vs. *F*S* contrast is helpful to classify past thinking in the field. A characteristic instance of a *D*W* approach to the problem of simple aimed movement is that of Woodworth (1899) [29], the celebrated pioneer, who explicitly held the variations of lengths *D* and *W* as causal. The essence of Woodworth's theory, still quite popular today [3,21], was that any simple aimed movement involves an initial open-loop phase, assumed to be selectively influenced by length *D*, followed by a closed-loop homing-in phase, assumed to be selectively influenced by length *W*.

A rather different view of the problem was put forward by Fitts [4], inspired by Shannon [24]. Relegating length measures *D* and *W* to the background, Fitts placed emphasis on the dimensionless quotient $Q_{D/W}$, on which he based his *ID* measurement. For good or bad reasons [16], Fitts' informa-

[7] Henceforth the term "quotient" will be distinguished from the term "ratio", whose meaning is equivocal. The quotient of a fractional expression is the *single* number one obtains by dividing the numerator by the denominator.

tion-theoretical approach was soon to be abandoned in subsequent basic research, though not in the HCI community. Fitts himself switched, in a decade's time, to a cognitive reformulation of his problem [5]. Nevertheless Fitts is still unanimously credited today for his empirical discovery that a certain log transform of the quotient $Q_{D/W}$ is a remarkably reliable predictor of μ_T.

Although Woodworth's and Fitts' writings offer sharply contrasted views of the simple aimed-movement problem, neither author was fully consistent. Woodworth hybridized his research with an occasional concern about the role of the Weber fraction *W/D*. Notice that such an quantity cannot be unequivocally tackled in his approach: having chosen to inquire into the influences of lengths *D* and *W*, Woodworth was not, for lack a third DOF, in a position to build any logically sound theory or to collect any conclusive data about the influence of task form, whether quantified from the quotient $Q_{D/W}$ or its inverse $Q_{W/D}$.

The problem with Fitts' account is his *incomplete F*S* approach. Having identified, thanks to Shannon's information theory, the special importance of the *F* factor in the simple aimed movement problem, Fitts did not realize that his analysis had to involve *S* as its second dimension, and it is apparent that subsequent authors have generally overlooked this other, perhaps less conspicuous DOF of expression *D/W*. So long as the logical necessity of defining the scale measure *S* as the complement of the form measure *F* is not recognized, it is hard not to hesitate between the two possible readings of Eq. 2. Such irresolution may explain why in basic and applied Fitts' law research experiments deliberately meant to reveal the effect of factor *F* are designed according to the alternative *D*W* logic.

Lack of coherence in the works of Woodworth, Fitts, and their successors up to present is no wonder if it is recognized that a fractional expression like *D/W*, the key causal entity of any Fitts' law equation, is inherently equivocal. What has been happening in Fitts' law research resembles what happens, in the sphere of human perception, to anyone invited to watch a Necker cube. Because there are two equally sensible ways of perceiving a 3D cube through that perfectly equivocal 2D picture, the visual system keeps on switching erratically from one view to the other. However, while the Necker cube bi-stability is an instructive trick of perception psychology, the conceptual bi-stability of the fractional expression *D/W* of Eq. 2 is a hindrance for the scientific study of simple aimed movement. One must choose one view to the exclusion of the other, explicitly and unequivocally. The next section aims to show that, insofar as optimal ranges of *S* are being considered, the main facts of the experimental literature strongly speak for the *F*S* description system.

THE *D*W* VERSUS *F*S* VIEW OF FITTS' LAW IN THE FACE OF GROSS EMPIRICAL EVIDENCE

To evaluate the understandings of the known facts that are permitted by the two mutually incompatible readings of Eq. 2, let us successively try the *D*W* viewpoint (Fig. 5) and the *F*S* viewpoint (Fig. 6). In this exercise the competing systems must be used uncompromisingly, with no reference made to factors *F* or *S* in the *D*W* analysis, and no reference to factors *D* or *W* in the *F*S* analysis. Also note that we will be focusing on gross qualitative evidence, the sort of evidence that no account could reasonably ignore. Technically, we will use a velocity vs. time description of simple aimed movement, which conveys more information than a mere chronometric description.[8]

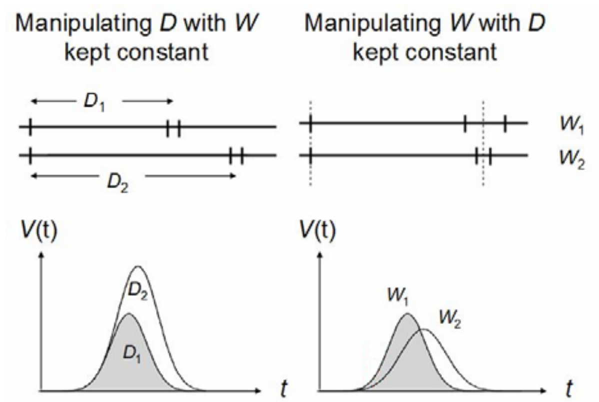

Figure 5. Above: the two basic factor manipulations consistent with the *D*W* description system. Below: schemas of how the movement's velocity vs. time profile is affected.

Fig. 5 illustrates schematically the effects that will take place quite inevitably if one manipulates the factors of the *D*W* system. An increase of target distance from D_1 to D_2, *W* being kept constant, will cause a lengthening of μ_T (Fig. 5, left), and so will a reduction of target width from W_1 to W_2, *D* being kept constant (Fig. 5, right). As anyone would expect, it takes more time to reach the target if it is farther and/or narrower. These two effects, both non-linear (with concave-down curvature for the effect of *D* and concave-up curvature for the effect of *W*), are strong but neither can be used as a simple predictor of μ_T. It would be tempting here to say that the interesting variable here is the ratio *D*/*W* (more accurately, the quotient $Q_{D/W}$), but recall that in the *D*W* analysis we have no third DOF available to define such a quantity independently of both *D* and *W*.

One very well documented observation is that the effect of selectively manipulating *W* will show up very early in the velocity vs. time profile of the movement (Fig. 5, right):

[8] In Figs. 5 and 6 movement time is simply the time elapsed between the zero-crossings of velocity at the beginning and end of the curve.

that W has been narrowed will be detectable (in the form of a reduction of acceleration) virtually from the start of the kinematical trace [1,14,15,6]. Such immediacy is rather disturbing for the $D*W$ approach, as it falsifies the widely shared hypothesis, inherited from Woodworth, that while D should affect mainly the initial impulse of the movement, W should affect mainly the terminal homing-in phase [29,3]. In sum, not only do the two basic manipulations of the $D*W$ system fail to deliver much insight, they also leave us with a worrying puzzle about the effect of factor W.

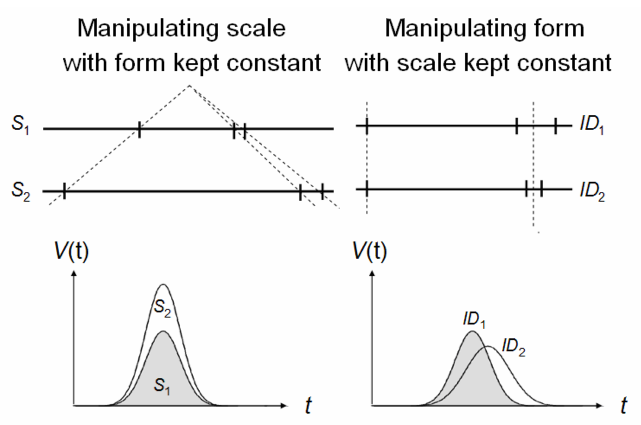

Figure 6. Above: the two basic factor manipulations consistent with the *F*S* description system. Below: schemas of how the movement's velocity vs. time profile is affected.[9]

Let us now switch to the alternative $F*S$ description system (Fig. 6). First one must ask what will happen if one rescales, from a small size S_1 to a larger size S_2, a task of a certain form, that is, with a certain quotient $Q_{D/W}$ and hence a certain level of ID (Fig. 6, left). It is known that, within limits, that manipulation will have little or no effect on μ_T. The velocity profile will present about the right amount of up-scaling to ensure that the increased distance will be covered in the same amount of time [30].[10] In and of itself, this time-conservation, or *isochrony* phenomenon constitutes a non-trivial finding that mere intuition could not have anticipated. Counting from Woodworth (1899) it actually took researchers fifty years to demonstrate that fact empirically, and today it is still overlooked in most accounts of Fitts' law. Second, one must ask what will happen if task form F is varied at a fixed level of scale S (Fig. 6, right): one will observe the explicit facet of Fitts' law, namely, μ_T will probably vary linearly with the logarithm of the quotient $Q_{D/W}$, as described in Eq. 1.

Notice that the form manipulation characteristic of the $F*S$ system (Fig. 6, right) is strictly the same, operationally speaking, as the manipulation of W in the $D*W$ system (Fig. 5, right). With D or S kept constant, it is just as correct to say that W (cm) is being reduced as it is to say that the dimensionless quotient $Q_{D/W}$ is being lowered. What differs between the two descriptions is the way the manipulation is construed: in the $D*W$ approach, which cannot recognize the quotient $Q_{D/W}$, it is the tolerance—a local, specific length measure—that is viewed to change. In the $F*S$ view, which cannot recognize the tolerance, it is the form or difficulty—i.e., a global, non-specific feature— of the task that is viewed to change. Both views would be tenable, but notice that the observation detail that looked so disturbing to the $D*W$ view (Fig. 5, right) is smoothly accommodated by the $F*S$ view: it looks quite natural that a change of the global form of the task causes a global and immediate reorganization of an aimed-movement (Fig. 6, right).

As argued in the preceding section, it is desirable to choose between the two candidate description systems and so it is good news that the main facts of the literature provide evidence in favor of one option. To recapitulate, it is easy to check, adopting the $D*W$ view, that both D and W exert potent influences on μ_T, but no elegant quantitative regularity has been identified along those lines. The $F*S$ enquiry, in contrast, ends up with the rewarding conclusion that one of the two independent variables, F, is always influential while the other, S, is usually not. Rather than two effects, we have a law of variation (μ_T varies lawfully with the logarithm of the quotient $Q_{D/W}$) along with a conservation law (μ_T is scale-independent). The variance of μ_T being under virtually exclusive control of one of the two DOF, the dependent variable can be predicted reliably from a single quantity. That an explicit formulation of both facets of the law becomes quite straightforward is an appreciable reward of recourse to the complete $F*S$ system. Presumably it is for lack of an unequivocal concept of scale that the important isochrony feature has generally received little attention in traditional Fitts' law research.

So logic and the empirical evidence jointly recommend the $F*S$ view for understanding the problem of simple aimed movement. But what is needed is a *complete* $F*S$ description, which considers both the form and the scale of the movement, and a *consistent* one, which ignores lengths D and W.

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[9] In this example task scale S is estimated by D, but it should not be confused with target distance. Varying S means varying *both* D and W proportionally, whereas varying D means varying D *but not* W.

[10] In a velocity vs. time profile the surface area under the curve specifies the distance, dimensionally $[L]*[T]^{-1} * [T] = [L]$. Incidentally, for a useful justification of dimensional analysis in movement research, see Hoffmann [12].

PRACTICAL DESIGN RECOMMENDATIONS FOR STANDARD FITTS' LAW EXPERIMENTATION

Re-Processing Data from the *D*W* Design

The technique shown in Fig. 3 (estimating the parameters of Fitts' law separately for each level of *D*, then averaging) may serve to correct a set of μ_T data collected with the usual *D*W* design. If the *S* factor, fortunately, had little or no influence on μ_T, as in Fitts' tapping experiment, the correction is superfluous. If, however, the data do show an effect of the *S* factor, as in Fitts' disc-transfer experiment (Fig. 3), the correction method may help to decontaminate the estimates of Fitts' law coefficients. The method cannot be entirely safe, however, because many data points are missing, and obviously recourse to the ANOVA is impossible. Thus it would seem preferable to preclude from the outset the possibility of any factor confound by designing fully consistent experiments. There are two ways of so doing: (1) using a complete *F*S* design or (2) eliminating the *S* variable altogether by keeping it constant at an appropriately chosen level.

The Difficulty of Implementing a Complete *F*S* Design

A target-acquisition experiment with reasonably narrow ranges of form and scale values (say, with the *ID* ranging between 2-8 bits [26] and *D* between 5-20cm) should produce a strong main effect of the *ID*, normally with no effect of scale (isochrony) and no interaction. However, boundary effects make it difficult to choose appropriate *ranges* for factors *F* and *S*, as illustrated in Table 2.

		Quotient $Q_{D/W}$ (-)						
		8	16	32	64	128	256	512
		Shannon *ID* (bit)						
		3.17	4.09	5.04	6.02	7.01	8.01	9.00
D (cm)	10.2	**1.270**	**0.635**	**0.318**	**0.159**	0.079	0.040	***0.020***
	20.3	2.540	**1.270**	**0.635**	**0.318**	**0.159**	0.079	0.040
	40.6	5.080	2.540	**1.270**	**0.635**	**0.318**	**0.159**	0.079
	81.3	***10.160***	5.080	2.540	**1.270**	**0.635**	**0.318**	**0.159**

Table 2. The cells of the design of Fitts' [4] disc-transfer experiment re-tabulated according to the *F*S* logic.

The seven values of *F* and the four values of *S* that Fitts [4] used in his disc-transfer experiment correspond to the columns and rows of Table 2 (to be compared with Table 1). Shown in the cells are the values taken by *W*, fully determined by the combinations of quotients and numerators of expression *D/W*. The 16 *F*S* conditions that Fitts actually used appear in bold, the 12 conditions he ignored in gray. Including in one's design all the 28 conditions of Table 2, one would get rid of the factor confounds of Fig. 1, but two serious technical difficulties arise. One is met toward the upper-right corner of the table, which wants both very difficult and very small movements, thus implying extremely narrow targets. Unless a zoom is available [7], an extreme case like *W* = 0.02cm is likely to be impractical in a pointing experiment. The other difficulty takes place in the opposite, lower-left corner region of the Table 2, where movement must be both very easy and very large. The problem here is that the isochrony law is unlikely to hold any more because that would require an excessive amount of velocity up-scaling.[11]

Experimental attempts with the *F*S* design should improve our general understanding of Fitts' law from a basic-research viewpoint [10]. However, given the technical difficulty of the form*scale design, that design seems hardly applicable in actual practice to routine Fitts' law evaluation experiments in HCI. Luckily enough, another just as consistent but far more economical design option is available.

Simple Task-Form Manipulations: An Economical Method of Estimating Fitts' Law Parameters

In HCI the main goal of a Fitts' law study is typically to estimate Fitts' law parameters for a number of devices or interaction techniques, so as to allow comparisons. Unless the researcher is specifically interested in scale effects, a safe and sensible strategy is to neutralize the scale factor by fixing it, and to focus on the all-important dependence of μ_T upon the *ID*, the task-form factor. In practice, using the circular array of targets recommended by MacKenzie and colleagues [26], the strategy will consist of choosing some comfortable diameter *D* once for all and just varying *W*. For a researcher interested in the evaluation of a number of competing techniques, the experimental design will thus involve just two factors, the technique factor and the *ID* (i.e., the task form factor), allowing a simple two-way analysis of variance. Not only is such a design conceptually preferable over the traditional three-way *D*W**technique design because it avoids the factor confounds of Fig. 1 as well as the perplexities of Fig. 3, it will also save experimental work as fewer conditions will have to be considered.[12]

Note that this simple design requires that movements of roughly optimal size be required of participants. Fitts' law, with its two facets (the scale independence as well as the lawful dependence of μ_T upon the *ID*), can hold only within an optimal range of scale levels and hence standard Fitts' law experiments should avoid the boundary effects that arise on either end of that range. Pragmatically, the scale

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[11] For example for *D* = 81.3cm in Fitts' Exp. 2, the best-fitting equation is $\mu_T = 0.064ID + 0.524$ ($r^2 = .997$). Extrapolation down to *ID* = 3.17bits (assuming strict isochrony) predicts $\mu_T = 0.73$s, hence an average velocity of over 1m/s (0.813m/0.73s), a rather demanding performance physically.

[12] This unconventional but simple design has been used systematically by this author in both basic, e.g. [6], and applied, e.g. [8], studies.

level should be judged too low, meaning the lower boundary has been reached, if target width *in and of itself* is a source of difficulty (e.g., a one-pixel target is usually hard to select however short the distance that has to be covered). In the up-scale direction, the scale level is probably too high if target distance is problematic *in and of itself* (i.e., if the required distance is hard to cover in a single move, however wide the target). A scale level about midway between these two boundaries should be quite safe.

TOWARD AN IMPROVED MATHEMATICAL DESCRIPTION OF FITTS' LAW AND OF ITS BREAKDOWN

It is this writer's contention that basic and applied research on simple aimed movement should benefit from recognition that the concepts of form (likely to be of relevance in the face of dimensionless quantities like $Q_{D/W}$) and scale are inseparable companions [27,19]. Explicit control over factor *S* in Fitts' law experimentation should reduce the amount of stochastic noise in the data and, one may hope, make it possible to eventually agree once for all on the mathematical description of the law. It is somewhat surprising that logarithmic [17], power [21], and linear [23] variants of Fitts' law equation have all survived fifty years of empirical testing. Perhaps the uncertainty we still have today concerning the mere factual description of Fitts' law, even in the favorable case of optimally-scaled movement, has to do with insufficient control thus far over data collection.

It would be imprudent to conclude from the foregoing analysis that the *F*S* approach to Fitts' paradigm is correct and the *D*W* approach just wrong. Even though, as argued above, the *F*S* description system seems most appropriate for tackling the case of optimally-scaled movement (i.e., where Fitts' law holds), the alternative description may well have promise for tackling the case of definitely under- and over-optimal movement scales (i.e., outside the limits of Fitts' law, where the law collapses).

Currently we are witnessing a spectacular development of drastically *miniaturized* as well as *enlarged* interfaces (e.g., handheld devices, interactive wall displays), and so in HCI research we urgently need a better understanding of what happens to pointing at non-optimal scales. A reasonable guess is that the downward or upward rescaling of any pointing task should lead at some point to an *absolute*-length problem—a too small *W* or a too large *D*, respectively—meaning the breakdown of Fitts' law and perhaps the need to switch to the alternative *D*W* description system. The elaboration and the experimental test of this possibility is a subject for future research.

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