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# On the Impact of Mobility on Outage Probability in Cellular Networks

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**Abstract**—In this paper, we develop an analytical study of the mobility in cellular networks and its impact on quality of service and outage probability. We first express analytically the downlink other-cell interference factor  $f$  by using a fluid model network. It is defined here as the ratio of outer cell received power to the inner cell received power. It allows us to analyze users mobility and to derive expressions of the outage probability. We show that mobility can modify the capacity of a cell and we quantify the outage probability variations. We moreover establish how mobility plays a role in quality of service management. All results are compared to Monte Carlo simulations performed in a traditional hexagonal network.

## I. INTRODUCTION

The estimation of cellular networks capacity mainly depends on the characterization of interference. In cellular networks such as CDMA or OFDMA systems, an important parameter for this characterization is the other-cell interference factor  $f$  (OCIF). The precise knowledge of the OCIF allows the derivation of outage probabilities, capacity evaluation and then, the definition of Call Admission Control mechanisms. It moreover plays a fundamental role for the analysis of mobility.

In this paper, we define OCIF as the ratio of total other-cell received power to the total inner-cell received power, though it is generally defined as the ratio of other-cell interference to inner-cell interference. On the downlink, [1] [2] aimed at computing an average OCIF over the cell by numerical integration in hexagonal networks. In [4], Gilhousen et al. provide Monte Carlo simulations and obtain an histogram of  $f$ . Chan and Hanly [5] precisely approximate the distribution of the other-cell interference. The modelling key of our approach is to consider the discrete BS entities of a cellular network as a continuum. Recently, the authors of [9] described a network in terms of macroscopic quantities such as the node density. The same idea is used in [10] for ad hoc networks. They however assumed a very high density of nodes in both papers and unlimited networks. We show hereafter that our model is accurate even when the density of BS is very low.

This paper extends the framework proposed in [6] and [7] that provides a simple closed-form formula for  $f$  on the downlink. We validate here the formulas by Monte Carlo simulations. Since OCIF characterizes the load generated by each mobile in a cell, we show that it is possible to get a simple outage probability approximation by integrating  $f$  over

a circular cell. We analyze moreover the impact of mobility on the outage probability.

We first introduce the interference model and the notations, then present the fluid model and its validation. Then, outage probability is analyzed. Afterwards, we derive analytical expressions which allow to analyze the impact that mobility can have on the management of the quality of service and on outage probability. In the last section, we present some numerical results.

## II. INTERFERENCE MODEL AND NOTATIONS

We consider a cellular system and we focus on the downlink. BS have omni-directional antennas, so that a BS covers a single cell. If a mobile  $u$  is attached to a station  $b$  (or serving BS), we write  $b = \psi(u)$ .

The propagation path gain  $g_{b,u}$  designates the inverse of the pathloss  $pl$  between station  $b$  and mobile  $u$ ,  $g_{b,u} = 1/pl_{b,u}$ .

The following power quantities are considered:

- $P_{b,u}$  is the transmitted power from station  $b$  towards mobile  $u$  (for user's traffic);
- $P_b = P_{cch} + \sum_u P_{b,u}$  is the total power transmitted by station  $b$ ,  $P_{cch}$  represents the amount of power used to support broadcast and common control channels.
- $p_{b,u}$  is the power received at mobile  $u$  from station  $b$ ; we can write  $p_{b,u} = P_b g_{b,u}$ ;
- $S_{b,u} = P_{b,u} g_{b,u}$  is the useful power received at mobile  $u$  from station  $b$  (for traffic data); since we do not consider soft handover, we can write  $S_u = S_{\psi(u),u}$ .

The total amount of power experienced by a mobile station  $u$  in a cellular system can be split up into several terms: useful signal ( $S_u$ ), interference and noise ( $N_{th}$ ). It is common to split the system power into two terms:  $p_u = p_{int,u} + p_{ext,u}$ , where  $p_{int,u}$  is the *internal* (or own-cell) received power and  $p_{ext,u}$  is the *external* (or other-cell) interference. Notice that we made the choice of including the useful signal  $S_u$  in  $p_{int,u}$ , and, as a consequence, it has to be distinguished from the commonly considered own-cell interference.

With the above notations, we define the interference factor in  $u$ , as the ratio of total power received from other BS to the total power received from the serving BS  $\psi(u)$ :  $f_u = p_{ext,u}/p_{int,u}$ . The quantities  $f_u$ ,  $p_{ext,u}$ , and  $p_{int,u}$  are location dependent and can thus be defined in any location  $x$  as long as the serving BS is known.

In downlink, orthogonality between physical channels may be approached by Hadamard multiplexing if the delay spread is much smaller than the chip duration  $T_c$ . As a consequence, a coefficient  $\alpha$ , may be introduced to account for the lack of perfect orthogonality in the own cell.

In this paper, we will use the signal to interference plus noise ratio (SINR) as the criteria of radio quality:  $\gamma_u^*$  is the SINR target for the service requested by MS  $u$ . This figure is a priori different from the SINR evaluated at mobile station  $u$ . However, we assume perfect power control, so  $SINR = \gamma_u^*$  for all users.

#### A. CDMA network

With the introduced notations, the SINR experimented by  $u$  can thus be derivated (see e.g. [3]):

$$\gamma_u^* = \frac{S_u}{\alpha(p_{int,u} - S_u) + p_{ext,u} + N_{th}} \quad (1)$$

From this relation, we can express  $S_u$  as:

$$S_u = \frac{\gamma_u^*}{1 + \alpha\gamma_u^*} p_{int,u} (\alpha + p_{ext,u}/p_{int,u} + N_{th}/p_{int,u}) \quad (2)$$

We denote  $\beta_u = \frac{\gamma_u^*}{1 + \alpha\gamma_u^*}$ . The transmitted power for MS  $u$ ,  $P_{b,u} = S_u/g_{b,u}$ , using relations  $p_{int,u} = P_b g_{b,u}$  and  $f = p_{ext}/p_{int}$  can be written as:

$$P_{b,u} = \beta_u (\alpha P_b + f_u P_b + N_{th}/g_{b,u}). \quad (3)$$

#### B. OFDMA network

In OFDMA, the data are multiplexed over a great number of subcarriers. There is no internal interference, so we can consider that  $\alpha(p_{int,u} - S_u) = 0$ . Since  $p_{ext,u} = \sum_{j \neq b} P_j g_{j,u}$ , we can write:

$$\gamma_u = \frac{P_{b,u} g_{b,u}}{\sum_{j \neq b} P_j g_{j,u} + N_{th}} \quad (4)$$

so we have

$$\gamma_u = \frac{1}{f_u + \frac{N_{th}}{P_{b,u} g_{b,u}}} \quad (5)$$

Since  $\frac{N_{th}}{P_{b,u} g_{b,u}} \ll f_u$ , typically for cell radii less than about 1 km, we can neglect this term and write

$$\gamma_u = \frac{1}{f_u} \quad (6)$$

### III. FLUID MODEL

In this section, we first present the model, derive the closed-form formula for  $f$ , and validate it through Monte-Carlo simulations in a hexagonal network.

#### A. OCIF formula

The key modelling step of the model we propose consists in replacing a given fixed finite number of BS by an equivalent continuum of transmitters which are spatially distributed in the network. This means that the transmitting power is now considered as a continuum field all over the network. In this context, the network is characterised by a MS density  $\rho_{MS}$  and a base station density  $\rho_{BS}$  [6] [8]. We assume that MS and BS are uniformly distributed in the network, so that  $\rho_{MS}$  and  $\rho_{BS}$  are constant. As the network is homogeneous, all base stations have the same output power  $P_b$ .

We focus on a given cell and consider a round shaped network around this centre cell with radius  $R_{nw}$ . The half distance between two base stations is  $R_c$  (see Figure 1).

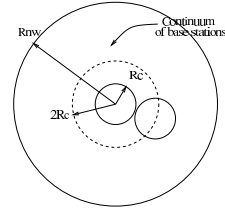


Fig. 1. Network and cell of interest in the fluid model; the distance between two BS is  $2R_c$  and the network is made of a continuum of base stations.

For the assumed omni-directional BS network, we use a propagation model, where the path gain,  $g_{b,u}$ , only depends on the distance  $r$  between the BS  $b$  and the MS  $u$ . The power,  $p_{b,u}$ , received by a mobile at distance  $r_u$  can thus be written  $p_{b,u} = P_b K r_u^{-\eta}$ , where  $K$  is a constant and  $\eta > 2$  is the path-loss exponent.

Let's consider a mobile  $u$  at a distance  $r_u$  from its serving BS  $b = \psi(u)$ . Each elementary surface  $zdzd\theta$  at a distance  $z$  from  $u$  contains  $\rho_{BS}zdzd\theta$  base stations which contribute to  $p_{ext,u}$ . Their contribution to the external interference is  $\rho_{BS}zdzd\theta P_b K z^{-\eta}$ . We approximate the integration surface by a ring with centre  $u$ , inner radius  $2R_c - r_u$ , and outer radius  $R_{nw} - r_u$  (see Figure 2).

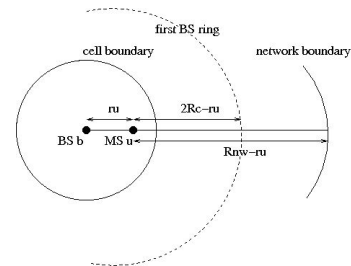


Fig. 2. Integration limits for external interference computation.

$$\begin{aligned} p_{ext,u} &= \int_0^{2\pi} \int_{2R_c - r_u}^{R_{nw} - r_u} \rho_{BS} P_b K z^{-\eta} z dz d\theta \\ &= \frac{2\pi \rho_{BS} P_b K}{\eta - 2} [(2R_c - r_u)^{2-\eta} - (R_{nw} - r_u)^{2-\eta}]. \quad (7) \end{aligned}$$

Moreover, MS  $u$  receives internal power from  $b$ , which is at distance  $r_u$ :  $p_{int,u} = P_b K r_u^{-\eta}$ . So, the interference factor  $f_u = p_{ext,u}/p_{int,u}$  can be expressed by:

$$f_u = \frac{2\pi\rho_{BS}r_u^\eta}{\eta - 2} [(2R_c - r_u)^{2-\eta} - (R_{nw} - r_u)^{2-\eta}]. \quad (8)$$

Note that  $f_u$  does not depend on the BS output power. This is due to the fact that we assumed an homogeneous network and so all base stations emit the same power. In our model,  $f$  only depends on the distance  $r$  to the BS and can be defined in each location, so that we can write  $f$  as a function of  $r$ ,  $f(r)$ . Thus, if the network is large, i.e.  $R_{nw}$  is big in front of  $R_c$ ,  $f_u$  can be further approximated by:

$$f(r) = \frac{2\pi\rho_{BS}r^\eta}{\eta - 2} (2R_c - r)^{2-\eta}. \quad (9)$$

This closed-form formula allows to fastly compute performance parameters of a cellular networks. However, before going ahead, we need to validate the different approximations we made in this model.

### B. Validation of the Fluid Model

In this section, we aim at validating the fluid model presented above. In this perspective, we will compare the figures obtained with Eq.8 with those obtained numerically by simulations. The simulator assumes an homogeneous hexagonal network made of several rings around a centre cell. Figure 3 shows an example of such a network with the main parameters involved in the study.

The fluid model and the traditional hexagonal model are two simplifications of the reality. None is a priori better than the other but the latter is widely used, especially for dimensioning purposes. That is the reason why a comparison is useful.

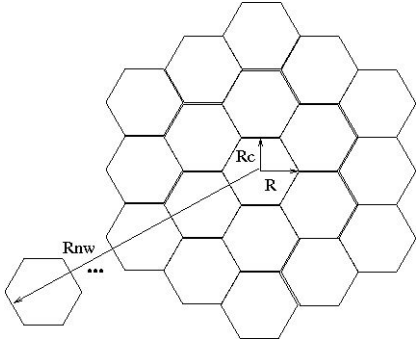


Fig. 3. Hexagonal network and main parameters of the study.

The validation is done numerically by computing  $f$  in each point of the cell and averaging the values at a given distance from the BS. This computation can be done independently of the number of MS in the cell and of the BS output power. Factor  $f$  indeed depends only on the path-losses to the BS of the network.

Figure 4 shows the simulated interference factor as a function of the distance to the base station. Simulation parameters

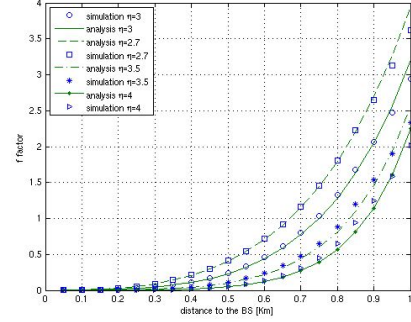


Fig. 4. Interference factor vs. distance to the BS; comparison of the fluid model with simulations on an hexagonal network with  $\eta = 2.7, 3, 3.5$ , and 4.

are the following:  $R = 1$  Km,  $\alpha = 0.7$ ,  $\eta$  between 2.7 and 4,  $\rho_{BS} = (3\sqrt{3}R^2/2)^{-1}$ , the number of rings is 15, and the number of snapshots is 1000. Eq.(8) is also plotted for comparison. In all cases, the fluid model matches very well the simulations on an hexagonal network for various figures of the path-loss exponent. It allows calculating the influence of a mobile, whatever its position in a cell.

We notice moreover that the fluid model can be used even for great distances between the base stations: We validated the model considering a distance of 2 Km between the BS. We conclude that our approach is accurate even for a very low base station's density.

## IV. OUTAGE PROBABILITIES

In this section, we express the outage probability. Though mainly focused on CDMA systems, our analysis can easily be extended to others such as OFDMA networks.

### A. Base station transmitting power

From Eq.(3), the output power of BS  $b$  can be computed as follows:

$$P_b = P_{cch} + \sum_u P_{b,u}, \quad (10)$$

and so, according to Eq.3,

$$P_b = \frac{P_{cch} + \sum_u \beta_u \frac{N_{th}}{g_{b,u}}}{1 - \sum_u \beta_u (\alpha + f_u)}. \quad (11)$$

The expression (11) shows that mobiles generate a load depending on the service used and the interference factor. Since the base station transmitting power is limited, and assuming a negligible Noise (which is reasonable as long as the cell's radius is less than about 1 km), that load, expressed as

$$L = \sum_{u=1}^M \beta_u (\alpha + f_u) \quad (12)$$

is also limited.

## B. Outage probability

For a given number of MS per cell,  $n$ , outage probability,  $P_{out}^{(n)}$ , is the proportion of configurations, for which the needed resources exceeds the available ones.

In particular for CDMA systems, this happens when BS output power exceeds the maximum output power:  $P_b > P_{max}$ . We deduce from Eq.11:

$$P_{out}^{(n)} = Pr \left[ \sum_{u=0}^{n-1} \beta_u (\alpha + f(r_u)) + \sum_{u=0}^{n-1} \beta_u \frac{N_{th}}{g_{b,u}} / P_{max} > 1 - \varphi \right], \quad (13)$$

where  $\varphi = P_{cch}/P_{max}$  and  $\beta = \gamma^*/(1 + \alpha\gamma^*)$ .

The parameter  $L$  defined as (14) represents the cell load

$$L = \sum_{u=0}^{n-1} \beta_u (\alpha + f(r_u)) + \sum_{u=0}^{n-1} \beta_u \frac{N_{th}}{g_{b,u}} / P_{max} \quad (14)$$

If noise is neglected, typically for cell radius less than 1 km, and if we assume a single service network ( $\gamma_u^* = \gamma^*$  for all  $u$ ), we can write

$$P_{out}^{(n)} = Pr \left[ \sum_{u=0}^{n-1} (\alpha + f(r_u)) > \frac{1 - \varphi}{\beta} \right], \quad (15)$$

## V. MOBILITY ANALYSIS

The configuration of a cellular system can vary with time. Indeed, since mobiles move, new ones begin a new call, and others end their call, mobiles distributions in a cell are different at two instants  $t$  and  $t_0$ . As a consequence, the load of a cell varies and the available bandwidth can be different at an instant  $t > t_0$ . We analyze hereafter how these variations may impact the management of quality of service (characterized here by the parameter  $\beta$  see sections V-A V-B V-C) and outage probability.

We consider two mobiles in the cell. Assuming no new call and no end of call, we can write the load time variation as (with low noise):

$$\begin{aligned} \sum_{i=1}^2 (\beta_i (\alpha + f_i)) (t) &= (\beta_1 (\alpha + f_1) + \beta_2 (\alpha + f_2)) (t_0) \\ &+ (t - t_0) \frac{d}{dt} [\beta_1 (\alpha + f_1) + \beta_2 (\alpha + f_2)]. \end{aligned} \quad (16)$$

The last derivation can be written

$$\begin{aligned} \frac{d}{dt} [\beta_1 (\alpha + f_1) + \beta_2 (\alpha + f_2)] &= \\ \left( \frac{\partial \beta_1}{\partial t} + \vec{v}_1 \cdot \frac{\partial \beta_1}{\partial \vec{r}} \right) (\alpha + f_1) + \beta_1 \left( \frac{\partial f_1}{\partial t} + \vec{v}_1 \cdot \frac{\partial f_1}{\partial \vec{r}} \right) &+ \\ \left( \frac{\partial \beta_2}{\partial t} + \vec{v}_2 \cdot \frac{\partial \beta_2}{\partial \vec{r}} \right) (\alpha + f_2) + \beta_2 \left( \frac{\partial f_2}{\partial t} + \vec{v}_2 \cdot \frac{\partial f_2}{\partial \vec{r}} \right) & \end{aligned} \quad (17)$$

Let's consider that only the mobile 1 moves. So we have  $v_1 \neq 0$  and  $v_2 = 0$ . Moreover we assume negligible time variations of the interference factor  $f_i$  such as it does not explicitly depend on the time, so we can write:

$$\begin{aligned} \frac{d}{dt} [\beta_1 (\alpha + f_1) + \beta_2 (\alpha + f_2)] &= \\ \left( \frac{\partial \beta_1}{\partial t} + \vec{v}_1 \cdot \frac{\partial \beta_1}{\partial \vec{r}} \right) (\alpha + f_1) + \beta_1 \vec{v}_1 \cdot \frac{\partial f_1}{\partial \vec{r}} + (\alpha + f_2) \left( \frac{\partial \beta_2}{\partial t} \right) & \end{aligned} \quad (18)$$

Considering the fluid model, the interference factor of a mobile located at the distance  $r$  from its serving BS can be written as a function of the distance  $r$  only,  $f_i = f(r)$ .

We denote  $\frac{\partial}{\partial t} \beta = \dot{\beta}$ ,  $\frac{\partial}{\partial r} \beta = \beta_{,r}$  and  $\frac{\partial}{\partial r} f = f_{,r}$ . The equation (18) can be rewritten

$$\begin{aligned} \frac{d}{dt} [\beta_1 (\alpha + f_1) + \beta_2 (\alpha + f_2)] &= \\ (\dot{\beta}_1 + v_1 \beta_{1,r}) (\alpha + f_1) + \beta_1 v_1 f_{1,r} + (\alpha + f_2) \dot{\beta}_2 & \end{aligned} \quad (19)$$

### A. Constant Load

In a first analysis we consider that at each instant the bandwidth is shared between two mobiles in the cell. To optimize the use of the available transmitting power of a base station, we write that the load has to be maximal:

$$\sum_{u=1}^M \beta_u (\alpha + f_u) = 1 - \varphi \quad (20)$$

If the power ratio dedicated to traffic channels is constant (which is a reasonable assumption in a real network) we can write from (20):

$$\beta_1 (\alpha + f_1) + \beta_2 (\alpha + f_2) = 1 - \varphi \quad (21)$$

We need to analyze how this expression varies with time. Since  $\varphi$  is a constant, the derivative of this expression to the time must be zero, which gives (from 19):

$$(\dot{\beta}_1 + v_1 \beta_{1,r}) (\alpha + f_1) + \beta_1 v_1 f_{1,r} = -(\alpha + f_2) \dot{\beta}_2 \quad (22)$$

The provider can have a policy consisting to allocate a constant SINR to some specific mobiles, all long the communication: According to that policy, we consider that the SINR of the mobile 2 is constant so we have  $\dot{\beta}_2 = 0$ . The expression (22) can thus be written:

$$\frac{\dot{\beta}_1 + v_1 \beta_{1,r}}{v_1 \beta_1} = -\frac{f_{1,r}}{\alpha + f_1} \quad (23)$$

We can moreover consider the two cases:

1)  $\dot{\beta}_1 = 0$ : This condition means that the SINR of mobile 1 is not an explicit function of time, so we have:

$$\frac{\beta_{1,r}}{\beta_1} = -\frac{f_{1,r}}{\alpha + f_1} \quad (24)$$

This case can correspond to a policy for which the provider does not impose a time variation of user 1 SINR. However, in the aim to use the whole available bandwidth, this one has to adapt his throughput according to his position in the cell. We moreover establish an interesting result from (24): the SINR spatial variations **do not depend** on the mobile speed  $v_1$ .

2)  $\beta_{1,r} = 0$ : This case can correspond to a policy for which the provider *imposes* that a mobile has the same SINR whatever his position in the cell. This SINR has however to vary with time in the aim to adapt itself when the mobile moves:

$$\frac{\dot{\beta}_1}{\beta_1} = -v_1 \frac{f_{1,r}}{\alpha + f_1} \quad (25)$$

### B. Constant Quality of Service

The provider can have a policy consisting to guarantee a constant quality of service all long the communication: According to that policy, we consider that the SINR are constant so we have  $\dot{\beta} = 0$  and  $\beta_{,r} = 0$ . The expression (19) can thus be written:

$$\frac{d}{dt} [\beta_1(\alpha + f_1) + \beta_2(\alpha + f_2)] = \beta_1 v_1 f_{1,r} \quad (26)$$

so we have from (16)

$$\begin{aligned} & (\beta_1(\alpha + f_1) + \beta_2(\alpha + f_2))(t) \\ &= (\beta_1(\alpha + f_1) + \beta_2(\alpha + f_2))(t_0) \\ & \quad + (t - t_0) \beta_1 v_1 f_{1,r}. \end{aligned} \quad (27)$$

### C. Variable Quality of Service

1)  $\dot{\beta}_i = 0$ : This condition means that mobiles SINR are not an *explicit* function of time. The expression (19) can be rewritten:

$$\frac{d}{dt} [\beta_1(\alpha + f_1) + \beta_2(\alpha + f_2)] = (v_1 \beta_{1,r})(\alpha + f_1) + \beta_1 v_1 f_{1,r}, \quad (28)$$

so we have from (16)

$$\begin{aligned} & (\beta_1(\alpha + f_1) + \beta_2(\alpha + f_2))(t) \\ &= (\beta_1(\alpha + f_1) + \beta_2(\alpha + f_2))(t_0) \\ & \quad + (t - t_0)(\beta_{1,r}(\alpha + f_1) + \beta_1 f_{1,r})v_1. \end{aligned} \quad (29)$$

This case can correspond to a policy for which the provider does not impose a time variation of mobiles SINRs. However, the SINR allocated to a mobile depends on its position in the cell.

2)  $\beta_{1,r} = 0$ : This condition means that mobile's 1 SINR is not an *explicit* function of its position. This case can correspond to a policy for which the provider *imposes* that a mobile has the same SINR whatever its position in the cell. This SINR can however vary with time. The equation (19) can be rewritten

$$\frac{d}{dt} [\beta_1(\alpha + f_1) + \beta_2(\alpha + f_2)] = (\alpha + f_1)\dot{\beta}_1 + \beta_1 v_1 f_{1,r} + (\alpha + f_2)\dot{\beta}_2. \quad (30)$$

so we have from (16)

$$\begin{aligned} & (\beta_1(\alpha + f_1) + \beta_2(\alpha + f_2))(t) \\ &= (\beta_1(\alpha + f_1) + \beta_2(\alpha + f_2))(t_0) \\ & \quad + (t - t_0) \left( (\alpha + f_1)\dot{\beta}_1 + \beta_1 v_1 f_{1,r} + (\alpha + f_2)\dot{\beta}_2 \right). \end{aligned} \quad (31)$$

### D. N mobiles analysis

We consider N mobiles in a cell, the expression (19) can be rewritten as

$$\sum_{i=1}^N \frac{d}{dt} [\beta_i(\alpha + f_i)] = \sum_{i=1}^N \left[ \left( \frac{\partial \beta_i}{\partial t} + \vec{v}_i \cdot \frac{\partial \beta_i}{\partial \vec{r}} \right) (\alpha + f_i) + \beta_i \vec{v}_i \cdot \frac{\partial f_i}{\partial \vec{r}} \right] \quad (32)$$

Considering that a  $(N + 1)^{th}$  mobile can enter at instant t, we have from (16)

$$\begin{aligned} \sum_{i=1}^{N+1} \beta_i(\alpha + f_i)(t) &= \sum_{i=1}^N \beta_i(\alpha + f_i)(t_0) + \beta_{N+1}(\alpha + f_{N+1})(t) \\ & \quad + (t - t_0) \sum_{i=1}^N \left[ \left( \frac{\partial \beta_i}{\partial t} + \vec{v}_i \cdot \frac{\partial \beta_i}{\partial \vec{r}} \right) (\alpha + f_i) + \beta_i \vec{v}_i \cdot \frac{\partial f_i}{\partial \vec{r}} \right]. \end{aligned} \quad (33)$$

### E. Application to outage probability analysis

Let consider a given number of MS per cell, N, at the instant  $t_0$ . If noise is neglected, the outage probability  $P_{out}^{(N)}$ , is given by the expression (15). When a  $(N + 1)^{th}$  mobile asks entering the cell at the instant  $t > t_0$ , the outage probability can be written (from (15)):

$$P_{out}^{(N+1)} = Pr \left[ \left( \sum_{u=1}^{N+1} \beta_u(\alpha + f(r_u)) \right)(t) > 1 - \varphi \right]. \quad (34)$$

Applying (33), we can write

$$\begin{aligned} P_{out}^{(N+1)} &= Pr \left( \left( \sum_{i=0}^N \beta_i(\alpha + f_i) \right)(t_0) + \beta_{N+1}(\alpha + f_{N+1})(t) \right. \\ & \quad \left. + (t - t_0) \sum_{i=1}^N \left[ \left( \frac{\partial \beta}{\partial t} + \vec{v}_i \cdot \frac{\partial \beta}{\partial \vec{r}} \right) (\alpha + f_i) + \beta_i \vec{v}_i \cdot \frac{\partial f_i}{\partial \vec{r}} \right]_{t=t_0} \right. \\ & \quad \left. > 1 - \varphi \right). \end{aligned} \quad (35)$$

*Constant quality of service:* We analyze hereafter the impact of mobility on outage probability when the policy consists to guarantee the quality of service. We have  $\dot{\beta} = 0$  and  $\beta_{,r} = 0$ . The expression (35) can thus be written:

$$\begin{aligned} P_{out}^{(N+1)} &= Pr \left( \sum_{i=1}^N \beta_i(\alpha + f_i)(t_0) + \beta_{N+1}(\alpha + f_{N+1})(t) \right. \\ & \quad \left. + (t - t_0) \sum_{i=1}^n [\beta_i v_i f_{i,r}]_{t=t_0} \right. \\ & \quad \left. > 1 - \varphi \right), \end{aligned} \quad (36)$$

The expression (36) means that at the instant t, the cell load variation is due to the entrance of an  $(N + 1)^{th}$  mobile (term  $\beta_{N+1}(\alpha + f_{N+1})(t)$ ) and the movement of the other mobiles already present in the cell (term  $(t - t_0) \sum_{i=1}^n [\beta_i v_i f_{i,r}]_{t=t_0}$ ). The first term is always positive. If the second term  $(t - t_0) \sum_{i=1}^n \beta_i v_i f_{i,r}$  is positive, the cell load increases and the

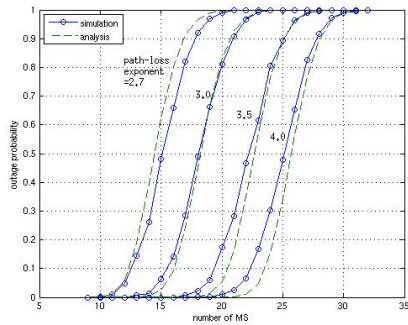


Fig. 5. Global outage probability as a function of the number of MS per cell and for path-loss exponents  $\eta = 2.7, 3, 3.5$  and  $4$ , simulation (solid lines) and analysis (dotted lines).

available resources decrease. As a consequence, the outage probability increases. This happens when the *mean radial speed* of mobiles  $v$  is positive and the mean interference factor increases. If the second term is negative, it may *compensate*, partly or even totally, the load due to the entrance of the  $(N + 1)^{th}$  mobile. In some cases, the outage probability may decrease.

## VI. NUMERICAL APPLICATION

We present simulations for  $v > 0$  and  $v < 0$  such as  $(t - t_0) \sum_{i=0}^{n-1} \beta_i v_i f_{i,r} = 0.1$  or  $(t - t_0) \sum_{i=0}^{n-1} \beta_i v_i f_{i,r} = -0.1$  and  $\eta = 2.9$  (Fig. 6). We calculate  $f_{i,r}$  using the expression of  $f(r)$  given by (9). We consider that all mobile use the same service  $\beta_i = \beta = \text{constant}$ . Figures 5 and 6 show the kind of results we are able to obtain instantaneously thanks to the simple formulas derived in this paper for voice service ( $\gamma_u^* = -16$  dB). Analytical formulas are compared to Monte Carlo simulations in an hexagonal cellular network. Figure 5 shows the outage probabilities as a function of the number of MS per cell for various values of the path-loss exponent  $\eta$ . It allows us to easily find the capacity of the network at any given maximum percentage of outage. For example, the outage probability when there are 12 users per cell is about 10% with  $\eta = 3.5$ .

Figure 6 shows the impact of mobility on the outage probability. Comparison of the left curve  $v > 0$  and the right curve  $v < 0$  to the central one  $v = 0$  allows to quantify with high accuracy the increase or decrease of outage probability due to mobility, when mobiles have a positive or negative mean speed. For example, when  $v = 0$  the outage probability is 14% for 16 mobiles and reaches 21% when a 17<sup>th</sup> mobile enters the cell. When  $v < 0$ , we observe that for 17 mobiles the outage probability is only 4%: mobiles speed decreases the outage.

## VII. CONCLUSION

This paper analyzes mobility in cellular networks and its impact on outage probability. Using a fluid model for expressing the long-range impact of neighboring cells allows us to establish a closed-form formula of the interference

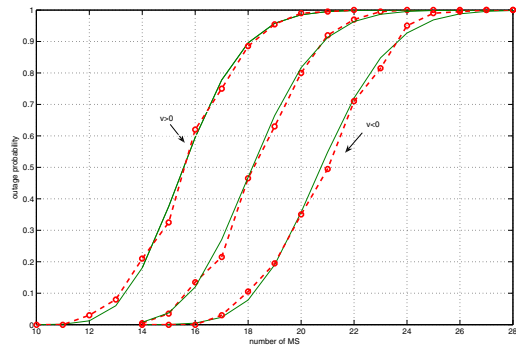


Fig. 6. Global outage probability as a function of the number of MS per cell and for path-loss exponents  $\eta = 2.9$ ,  $v > 0$  and  $v < 0$ , simulation (blue curves) and analytical (red dotted curves) approaches.

factor  $f$ . We establish its fundamental role on the analysis of outage probability and mobility. We show that mobility can play a role in management of users quality of service. We moreover show how mobility can modify the capacity of a cell, and quantify this modification. All results are compared to Monte Carlo simulations performed in a traditional hexagonal network. Though outage probability analysis is mainly focused on CDMA networks, that approach can easily be extended to OFDMA systems. In future works, we aim to analyze with more details some cases developed in this paper, such as the impact of different policies.

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