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Receiver study for cooperative communications in convolved additive α -stable interference plus Gaussian thermal noise

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Abstract—In wireless *ad hoc* network communications, both the network interference and the thermal noise should be considered in receiver design, due to the strong impairments each may cause on the quality of the reception at the destination. Since the closure under convolution of stable distributions only holds for the same stability index members, in general the additive convolution of the impulsive stable interference and lighter tailed Gaussian thermal noise will not result in a stable pattern. It is therefore a challenge to adequately model the distribution of such a process. In this context we consider an optimal receiver design and develop an importance sampling approach to perform estimation of the optimal receiver in the presence of convolved stable and Gaussian noises. Such an approximation approach to the optimal receiver is computationally expensive, hence we also develop as comparisons several suboptimal realizations of linear and non-linear receivers, including an approximation approach based on the Normal Inverse Gaussian (NIG) distribution. We demonstrate that the computationally efficient NIG receiver provides an alternative solution for the optimal receiver approximation. In addition we show that the p -norm receiver appears to have robust performance no matter what kind of noise is dominant.

Index Terms—*ad hoc* network, α -stable distribution, importance sampling, NIG distribution, optimal receiver, p -norm

I. INTRODUCTION

We consider a wireless *ad hoc* network in which communications are realized by decode-and-forward (DF) technology with the cooperation of multiple relays. We investigate a two-hop network, consisting of one source, N possible relays and one destination. According to [1], communications impairments in *ad hoc* networks can be divided into wireless propagation effects, network interference and thermal noise. For the propagation effects, we consider a slow-fading channel, for which the channel coefficients are constant for each time slot and change independently from one time to another. The accumulation of undesired signals from other nodes creates the network interference, which significantly disturbs the communications, increasing error probability for receivers. The α -stable distributions have been successfully introduced for interference modeling in several contexts [1]–[3]. We consider the sub-family of symmetric stable models to capture the

effects of network interference. Additional to and independent from this interference is the thermal noise at the destination, caused by receiver equipment, which is commonly modeled as a Gaussian distribution. Hence, the combined noise is captured by the convolution between an independent, symmetric stable distributed network interference and a Gaussian thermal noise, the result of which is not in general stable distributed.

When considering the optimal receiver in this context, one faces a challenge in the design of an efficient solution due to its inherent intractability, arising from the convolution between network and thermal noises. Some proposed receivers [4]–[6] give feasible but complex solutions. We give in our paper some ideas for designing either optimal or suboptimal receivers which are efficient alternatives, and we provide a global comparison of the proposed approach. Our contributions involves the following:

- development of a careful study of the decision strategy in a cooperative communication in the presence of non Gaussian network interference and thermal noise;
- we propose an original strategy based on developing a Normal Inverse Gaussian (NIG) receiver and compare this to a novel adaptation of the p -norm strategy, noting that the p -norm has already been proposed in other contexts, especially with generalized Gaussian distributions [7];
- we study the performance of the two suboptimal strategies as a function of the noise-to-interference ratio, comparing with the optimal receiver which is computationally inefficient but studied through importance sampling Monte Carlo estimation strategy, with linear receiver, hole puncher and soft limiter.

The p -norm and the NIG appear to be attractive solutions. The first one does not require any noise parameter estimation. The second one has flexible and efficient Moment Matching based closed form solutions for its parameter estimation.

The paper is organized as follows. We describe our system scenario and justify the use of symmetric α -stable (S α S) distributions for network interference in Section II. In Section III, we study the optimal receiver in the form of log-likelihood

ratio (LLR) and then consider some suboptimal solutions involving both linear and non-linear approaches. The NIG distribution is introduced and the Method of Moments based approximation approach is given, with analytical solutions after restriction to a symmetric sub-family, with no restriction required for the kurtosis. Finally we give the comparison of these techniques in simulation results, introducing meanwhile the importance sampling approach for the optimal decision. We provide a conclusion in Section V.

II. PROBLEM MODEL

A. System Scenario

We propose the following scenario: a set of K relays is selected among N possibilities. The selected relays are the ones with the strongest relay-destination channel. We assume that each selected relay decodes the signal sent from the source without error before transmitting it to the destination, all with the same transmit power. Relay-to-destination transmissions are made on orthogonal channels, and the synchronization is ideal with corrected phase. The received signal at the destination at time t is then given by:

$$\mathbf{y} = \mathbf{h}x + \mathbf{i} + \mathbf{n}, \quad (1)$$

where x is the transmitted signal from the source, and

- $\mathbf{h} = (h_0, h_1, \dots, h_K)$ are the channel coefficient vectors (index 0 denotes the direct link); they are Rayleigh distributed and, for a transmitted power equal to one, the received power expectation is $\mathbb{E}[|h_i|^2] = 1$;
- $\mathbf{i} = (i_0, i_1, \dots, i_K)$ is the independent and identically distributed (i.i.d.) interference, discussed in the next section.
- $\mathbf{n} = (n_0, n_1, \dots, n_K)$ is the i.i.d. thermal noise with $n_i \sim \mathcal{N}(0, \sigma^2)$.

B. Interference Model

In many papers [1]–[3], the S α S distributions family is used to model the interference in sensor networks in which power control is absent. The heavy tailed property of the stable model lends itself well to an accurate mathematical model for the infrequent impulsive or shot noise characteristics of the network interference. One can define the stable distribution used to model the network interference by its characteristic function. For the symmetric case (symmetry index $\beta = 0$) used in this paper, it is given by: $\phi(x) = \exp(-\gamma|x|^\alpha)$, in which only two parameters are required. The characteristic exponent α measures the index of regular variation or thickness of the distribution tail ($0 < \alpha < 2$). The smaller the value of α , the heavier the tail of the density becomes hence the more likely that a value appears far from the center location. The dispersion parameter γ is a scale parameter, similar to the variance of the Gaussian distribution [8]. As the PDF is the inverse Fourier transform of the characteristic function, a general stable distribution has no explicit expression for the density except for the Gaussian ($\alpha = 2$) and the Cauchy ($\alpha = 1$) cases [8].

III. RECEIVER STRATEGIES

A. Optimal Receiver

The detection problem for a binary source (i.e. $x \in \{s_0, s_1\}$) in the presence of stable network interference plus independent Gaussian thermal noise can be formally specified through a statement of a hypothesis test as

$$\begin{cases} H_0 : \mathbf{y} = \mathbf{h}s_0 + \mathbf{i} + \mathbf{n} \\ H_1 : \mathbf{y} = \mathbf{h}s_1 + \mathbf{i} + \mathbf{n}. \end{cases} \quad (2)$$

Given the transmitted binary symbols s_1 and s_0 and the observed received signal y , we define $\mathbb{P}_{i+n}(y|s_1)$ and $\mathbb{P}_{i+n}(y|s_0)$, where $\mathbb{P}_{i+n}(\cdot)$ represents the intractable probability distribution function obtained from the convolution between the stable network interference plus independent Gaussian thermal noise.

In the sense of minimizing the bit error rate (BER), the Bayes optimal receiver is employed. Assuming that the binary symbols are sent with equal probability, we have the *a priori* decision statistic in the form of LLR as:

$$\begin{aligned} \Lambda &= \log \frac{\mathbb{P}_{i+n}(\mathbf{y}|\mathbf{h}s_1)}{\mathbb{P}_{i+n}(\mathbf{y}|\mathbf{h}s_0)} \\ &= \log \frac{\prod_{k=1}^K f_{i+n}(y_k|h_k s_1)}{\prod_{k=1}^K f_{i+n}(y_k|h_k s_0)} \\ &= \sum_{k=1}^K \log \frac{f_{i+n}(y_k|h_k s_1)}{f_{i+n}(y_k|h_k s_0)} \underset{H_0}{\overset{H_1}{\gtrless}} 0, \end{aligned} \quad (3)$$

where k indicates the k^{th} relay-to-destination channel, and $f_{i+n}(\cdot)$ is the density of the interference plus noise. The decision between both hypothesis is made by comparing the LLR to the threshold 0.

B. Linear Receiver

The linear receiver is designed by assuming the density used in the decision statistics (3) is the Gaussian case (stable density when $\alpha = 2$). The linear receiver is also known as the Gaussian receiver and it is thus optimal when the interference is Gaussian distributed. The corresponding decision statistic is:

$$\begin{aligned} \Lambda_{\text{linear}} &= \sum_{k=1}^K \log \frac{f_2(y_k|h_k s_1)}{f_2(y_k|h_k s_0)} \\ &= \sum_{k=1}^K \log \frac{\exp[-(y_k - s_1)^2/2\sigma^2]}{\exp[-(y_k - s_0)^2/2\sigma^2]} \\ &= \frac{1}{\sigma^2} \sum_{k=1}^K y_k (s_1 - s_0) \underset{H_0}{\overset{H_1}{\gtrless}} 0, \end{aligned} \quad (4)$$

where σ is the standard deviation of Gaussian distribution. We will test such a receiver in the environment of stable ($\alpha < 2$) interference plus Gaussian noise. We primarily consider this choice for its simple implementation structure, though we predict that it will perform poorly when $\alpha \ll 2$.

C. Linear Combiner

An alternative linear solution is developed by consideration of the maximal ratio combiner (MRC), which is also simple to implement as a suboptimal receiver. The MRC has its original output form given by

$$\Lambda_{\text{MRC}} = \sum_{k=1}^K w_k y_k = \hat{\mathbf{s}} + \hat{\mathbf{n}}, \quad (5)$$

where $\mathbf{w} = \{w_k\}_{k=1}^K \in \mathbb{R}^K$ are the combiner weights, $\hat{\mathbf{s}}$ and $\hat{\mathbf{n}}$ are the weighted signal components and noise components.

The conventional MRC is optimal for independent Gaussian channels, for which the optimal weights are $w_k = h_k^*$, where $*$ represent the complex conjugate.

However, for detection in a stable interference plus Gaussian noise environment, the combiner has to take into account the interference parameters. An adapted optimal MRC is proposed in [9], [10], which provides the corresponding weights when only the S α S interference is present:

$$\begin{cases} w_k^* = \text{sign}(h_k) |h_k|^{1/(\alpha-1)}, & 1 < \alpha \leq 2 \\ w_j^* = \text{sign}(h_j), w_k^* = 0 \quad \forall k \neq j, & 0 < \alpha \leq 1 \end{cases} \quad (6)$$

for an arbitrary j in $\mathbf{i} = \arg\{|h_i| = \max\{|h_1|, \dots, |h_K|\}\}$.

D. Non-linear Receivers

1) *Cauchy Receiver*: As one special case of S α S distribution, Cauchy distributions ($\alpha = 1$) have their PDF with dispersion γ and median δ :

$$f_1(x) = \frac{\gamma}{\pi[\gamma^2 + (x - \delta)^2]}. \quad (7)$$

Cauchy receiver arose originally from the assumption that the tail index $\alpha = 1$ and should be optimal for the signal detection under pure Cauchy noise. By employing *a priori* decision statistic, we have Cauchy receiver as:

$$\begin{aligned} \Lambda_{\text{Cauchy}} &= \sum_{k=1}^K \log \frac{f_1(y_k | h_k s_1)}{f_1(y_k | h_k s_0)} \\ &= \sum_{k=1}^K \log \frac{\gamma^2 + (y_k - h_k s_0)^2}{\gamma^2 + (y_k - h_k s_1)^2} \underset{H_0}{\overset{H_1}{\geq}} 0. \end{aligned} \quad (8)$$

2) *Hole-puncher and Soft-limiter Receivers*: The Cauchy receiver presents two problems: the need to determine the parameter γ and the complexity to evaluate (8). A first idea is to add some non-linearity to the Gaussian receiver to limit the impact of large interference values. As proposed for instance in [8], [11], the hole-puncher and soft-limiter are commonly used non-linear functions. We use in our test these two functions with their forms as:

$$g_{\text{hp}}(x) = \begin{cases} x, & |x| < \kappa \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

and

$$g_{\text{sl}}(x) = \begin{cases} -\kappa, & x < -\kappa \\ x, & |x| < \kappa \\ \kappa, & x > \kappa \end{cases}, \quad (10)$$

where $g_{\text{hp}}(\cdot)$ and $g_{\text{sl}}(\cdot)$ replace $f_{i+n}(\cdot)$ in (3).

3) *p-norm Receiver*: It is noted that in the decision statistics (4) for the linear receiver, the metric used in the second step is the Euclidean distance between the received signal and the possible transmitted symbols. A corresponding metric exists in S α S case which measures the p^{th} order moment of the difference of two variables, noted as p -norm and is given for $0 < p < \alpha$ as:

$$\|X - Y\|_p = \begin{cases} [\mathbb{E}|X - Y|^p / C(\alpha, p)]^{1/p}, & 1 \leq \alpha \leq 2 \\ [\mathbb{E}|X - Y|^p / C(\alpha, p)]^{\alpha/p}, & 0 < \alpha < 1, \end{cases} \quad (11)$$

where $C(\alpha, p) = \frac{2^{p+1} \Gamma((p+1)/2) \Gamma(-p/\alpha)}{\alpha \sqrt{\pi} \Gamma(-p/2)}$, and $\Gamma(\cdot)$ is the gamma function.

This approach is of interest as it does not depend on any estimation of distribution parameters and a rough knowledge of α can be sufficient. The final decision is given by:

$$\Lambda_{p\text{-norm}} = \sum_{k=1}^K (|y_k - h_k s_0|^p - |y_k - h_k s_1|^p) \underset{H_0}{\overset{H_1}{\geq}} 0. \quad (12)$$

E. NIG approximation

From the hyper-geometric family of flexible skew-kurtosis models, the Normal Inverse Gaussian (NIG) distributions have analytical expressions for the probability density and first four moments in terms of the model parameters. This family of statistical models includes the Gaussian and Cauchy distributions as special limiting cases [12]. It is therefore of great interest to use this distribution to approximate our intractable PDF in the decision statistics.

The NIG model takes its name from the fact that it represents a Normal variance-mean mixture that occurs as the marginal distribution for a random variable Y when considering a pair of random variable (Y, Z) when Z is distributed as an inverse Gaussian $Z \sim \mathcal{IG}(\delta, \sqrt{\alpha^2 - \beta^2})$, and Y conditional on Z is $(Y|Z = z) \sim \mathcal{N}(\mu + \beta z, z)$. The resulting density function for the NIG model is given by:

$$f_{\text{NIG}}(y; \alpha, \beta, \mu, \delta) = \frac{\alpha \delta \exp[g(y)]}{\pi h(y)} K_1[\alpha h(y)], \quad (13)$$

where $K_1(\cdot)$ is a modified second kind Bessel function with index 1, $g(y) = \delta \sqrt{\alpha^2 - \beta^2} + \beta(y - \mu)$, and $h(y) = [(y - \mu)^2 + \delta^2]^{1/2}$.

The parameters have the constraints $\mu \in \mathcal{R}, \delta > 0, 0 \leq |\beta| \leq \alpha$. The parameter α is inversely related to the heaviness of the tails, where a small α corresponds to heavy tails that can accommodate outlying observations. The skewness is directly controlled by the parameter β , and $\beta = 0$ is the symmetric model. The location of the distribution is given by the parameter μ and the scale of the distribution is measured by the parameter δ .

We consider in our case a symmetric NIG model, which implies $\beta = 0$. We note the closed-form expressions for the mean, variance, skewness, and kurtosis of the NIG model as:

$$\begin{aligned} \mathbb{E}[y_k] &= \mu = h_k x; & \text{Var}[y_k] &= \frac{\delta}{\alpha}; \\ \text{Skew}[y_k] &= 0; & \mathbb{K}\text{urt}[y_k] &= \frac{3}{\delta \alpha}. \end{aligned}$$

In this way, the probability density for each link can be approximated by the estimated closed-form expressions from the observed values.

IV. SIMULATION RESULTS

A. Importance Sampling for Optimal Receiver

The optimal receiver depends on the value of (3). One method is the numerical computation, as the PDF is the inverse Fourier transform of the characteristic function:

$$f_{i+n}(x) = \frac{1}{\pi} \int_0^{+\infty} \phi_{i+n}(t) \cos(xt) dt, \quad (14)$$

where $\phi_{i+n}(t) = \exp(-\gamma|t|^\alpha - \sigma^2 t^2/2)$ is the interference plus noise characteristic function. This can be realized by discrete Fourier transform, but a heavy computation cost is inevitable for each channel realization and fixing the discrete step and truncation remains difficult. We utilise the importance sampling (IS) approach in our paper to calculate $f_{i+n}(\cdot)$.

IS was introduced in [13] as an efficient technique in the reduction of variance in random sampling, for it concentrates on the sample points where the value of the function is large. This approach can also be used for the simulation of rare random events, and for the generation of samples under a distribution which is difficult to generate directly [14].

In our context, the calculation for each $f_{i+n}(\cdot)$ is directly intractable, but the generation of S α S random variables is trivial. We can sample the interference component \mathbf{i} upon each value of \mathbf{h} and \mathbf{n} . Hence for one channel realization, y_k can be considered as under a normal distribution with the mean $h_k x + i_k$ and the variance of n_k (σ^2), which can be calculated as:

$$f_{i+n}(y_k | h_k x) = \int_I I(t) f_n(t) f_i(t) dt, \quad (15)$$

where I represents the interference sampling space, $f_n(\cdot)$ is the normal distribution PDF.

In the simulation, a number of M i.i.d. interference samples \mathbf{i}_k are generated under $f_i(\cdot)$, and the weighting function is defined as $\mathcal{N}(h_k x + i_k, \sigma^2)$, thus we have

$$\hat{f}_{i+n}(y_k | h_k x) = \frac{1}{M} \sum_{m=1}^M I_{k_m} \mathcal{N}(h_k x + i_{k_m}, \sigma^2). \quad (16)$$

As soon as the probability term $f_{i+n}(\cdot)$ is calculated, the decision of the optimal receiver can be made.

B. Comparison of different Strategies

We present in our simulation three different noise-to-interference ratios ($\sigma^2/2\gamma$) to investigate the described receivers. This ratio reflects different noise dominating environments, and we generated 500 noise samples for illustration in Figure 1.

We chose $K = 2$ strongest relay-to-destination channels among $N = 5$ possible ones. The optimal receiver is realized by IS approach as a benchmark, with 10^6 interference samples. The threshold for the hole-puncher receiver is set as $\kappa = 4$ and for the soft-limiter receiver as $\kappa = 1$. An empirical approach based

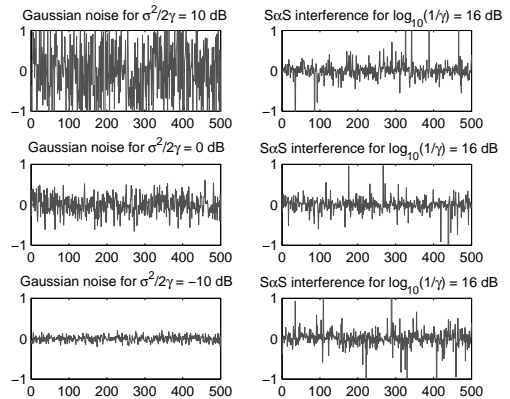


Fig. 1. Comparison of different noise dominating environments

on simulations was used to make choice for those parameters. We set $p = 0.8$ as an example value for all the simulations. Their performance is measured by BER in terms of the inverse value of dispersion of S α S interference in logarithm, since the increasing of the inverse dispersion implies the decreasing of the interference strength.

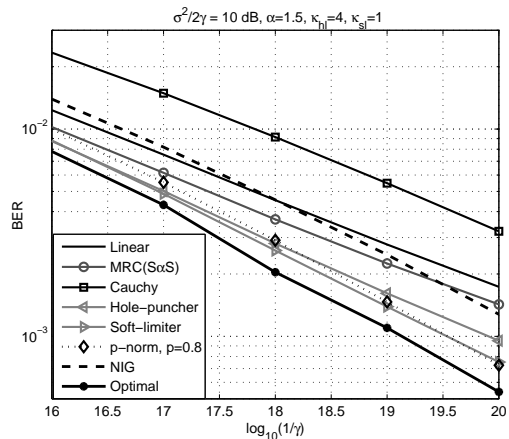


Fig. 2. Comparison of receivers in Gaussian-noise-dominant environment

In Figure 2, $\sigma^2/2\gamma$ is set to 10 dB which indicates that the dominant noise is Gaussian. We observe that the Cauchy receiver gives the worst BER, since it is optimal for Cauchy noise ($\alpha = 1$). The linear receiver and MRC give the same trend and the latter one is better for its adapted parameters. The NIG approximation shows similar performance as the linear approaches, because during the estimation of NIG parameters, the rare S α S interference parts have little influence on the dominant Gaussian noise. Hence the obtained NIG density is close to the limiting Gaussian case of this family. The p -norm, hole-puncher and soft-limiter receivers have almost same performance close to the optimal receiver. In case when the Gaussian noise and the S α S interference are comparable ($\sigma^2/2\gamma = 0$ dB), we can see in Figure 3 that the linear receiver and MRC are less capable to deal with the interference than the others. The Cauchy receiver exhibits a good performance

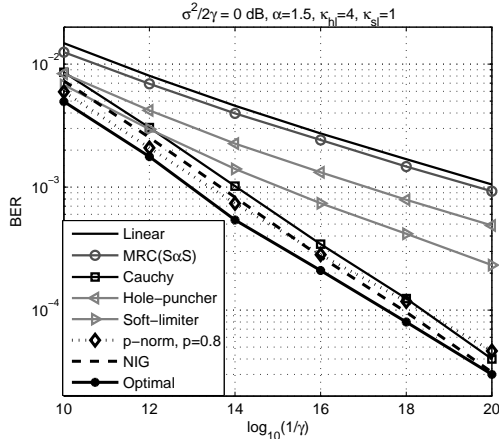


Fig. 3. Comparison of receivers in Gaussian-stable-comparable environment

in this condition, even surpassing the hole-puncher and soft-limiter. The p -norm receiver keeps close to the optimal case, while the NIG approximation gives similar performance as the p -norm approach. In Figure 4, when the $S\alpha S$ interference

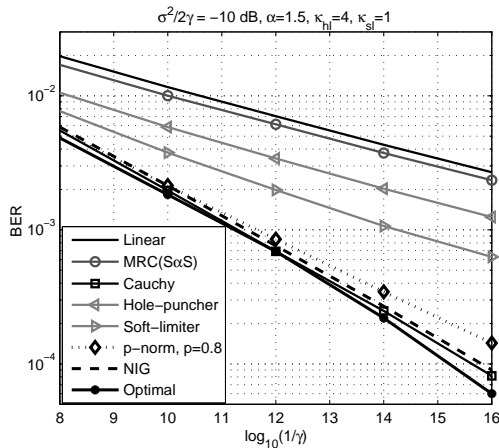


Fig. 4. Comparison of receivers in stable-interference-dominant environment

dominates the whole noise, the Cauchy receiver gives the best performance, close to the optimal. The p -norm and NIG receivers remain very close as well. The other receivers give significantly degraded performance.

From the above results, we can draw some conclusions:

- The linear receiver and MRC will have good performance only when the Gaussian noise is dominant.
- The Cauchy receiver performs well except when the Gaussian noise dominates.
- The hole-puncher and soft-limiter are good choices if the thresholds are well configured, but they are still far from optimality when impulsiveness is strong.
- The NIG approximation and the p -norm receiver have very good performance in all environment and approach very closely the optimal receiver when impulsiveness increases. It is noted that the p -norm has the best behavior when Gaussian noise is dominant and is a non-parametric

approach (at most a rough estimation of α is sufficient).

V. CONCLUSION

We evaluated the performance of several receivers in cooperative communications where both the network interference and the thermal noise are present. The $S\alpha S$ and Gaussian distributions were used to model the network interference and the thermal noise respectively. The chosen relays are the ones having the minimum relay-to-destination channel loss among a set of candidates. We proposed an importance sampling approach for the calculation of optimal receiver and several suboptimal receiver approaches. The simulation results showed that some parametric designs like the hole-puncher and soft-limiter receivers can have good performance if their thresholds are well configured. The NIG approximation proves to be a very efficient approach. The p -norm exhibits robust performance, no matter which noise is dominant. This gives us a simple and feasible strategy, which does not necessitate any noise parameter estimation.

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