Clipping demapper for LDPC decoding in impulsive channel

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***Abstract*—This paper deals with the performance of Low-Density Parity-Check codes in impulsive interference modeled by α-stable random variables. In case of α-stable noise, the optimal inputs of the belief propagation decoder are complex to obtain. We propose to use the simple clipping approach that reduces the impact of large noise values. Our main contribution is to give three different approaches to obtain the parameters of the clipping function and to assess the performance of the decoder. We show that a look-up table whose values are pre-determined, thanks to the Density Evolution tool, is the most efficient approach.**

***Index Terms*—LDPC codes, α-stable interference, density evolution, clipping.**

I. Introduction

DENSER deployment of wireless networks makes interference the main limitation to the system performance. Despite the limits due to the unbounded path-loss model assumption [1], an α-stable distribution can give a relevant insight and a powerful tool to analyze the effect of this aggregate interference, for instance in wireless *ad hoc* or sensor networks[2], [3].

As packet retransmissions induce an important energy dissipation, we suggest to use an efficient channel coding scheme to reduce them. We are interested in Low Density Parity Check (LDPC) codes for their good performances and their iterative decoding. However, the decoding process is particularly sensitive to the noise model and, if the noise is impulsive, performance is significantly degraded when receivers based on belief propagation (BP) are designed under Gaussian noise assumption [4].

As the LDPC decoder performance strongly depends on its input, the likelihood ratio to transform the received symbols must be carefully calculated in order to obtain a satisfactory complexity-performance trade-off. The design of optimal or suboptimal receivers is extensively studied in literature, *e.g.* in [5] for RAKE combiner and the scope of the paper is to focus on the simple and easy to implement clipping approach. This choice is justified by the necessity to clip the noise pulses and by the difficulty to implement the natural demapper. Our contribution is to study different strategies to obtain the different parameters of the clipping demapper —its slope and its threshold,— which highly influence the performance of the decoding algorithm.

Section II presents the model of the system; Section III deals with the optimization of the clipping demapper. Some simulations illustrate the performance obtained in Section IV before the conclusion.

II. System Model

A. Impulsive noise channel

We assume the model $Y = X + N$. Input X belongs to a simple BPSK constellation, $+1$ or -1. As mentioned in the introduction, N is an i.i.d. noise following symmetric α-stable distribution. Many papers have proposed justification for such a model [2] and we do not discuss it in this letter. The model is well adapted to several contexts when the noise is impulsive. It can be the case in *ad hoc* networks but also, for instance, in power line communications [6].

This distribution is stable in the sense that the sum of stable random variables is still stable [7]. Unfortunately, most often, no closed-form of its probability density function (pdf) is available. Nevertheless, we can express the characteristic function of symmetric α-stable distribution by $\varphi_N(t) = \mathbb{E}\left[e^{itN}\right] = e^{-|\gamma t|^\alpha}$.

This SαS distribution depends on two parameters: the exponent, or index, α and the scale factor γ. In wireless context, α is directly associated with the pathloss exponent of the radio channel [2]. The noise is all the more impulsive as α is closer to 1. The scale parameter γ measures the spread of the noise. The special case of Gaussian and Cauchy distributions are given by $\alpha = 2$ and $\alpha = 1$ respectively.

The n-th moment of a non-Gaussian α-stable distribution is finite only for $n < \alpha$. Thus, if $\alpha < 2$, the power is infinite and the signal to noise ratio is meaningless. Moreover, comparison of the scale parameters between two systems is meaningful only when they share the same index α. Several papers deal with this aspect and propose other "power" measurements [8]. To make things simple we will only consider γ as a measurement of the strength of the noise.

In order to give an exact impact of impulsiveness, we assume that no additive thermal noise contaminate the channel. We call this special case Additive Independent Stable Noise (AISN) channel.

B. Channel capacity

Due to symmetry, the capacity of the channel is the mutual information between the binary input and the noisy output of the channel. As generating α-stable samples is easy [7], a Monte-Carlo method can be used to numerically compute it [4]. Other more general methods may be considered [9] but

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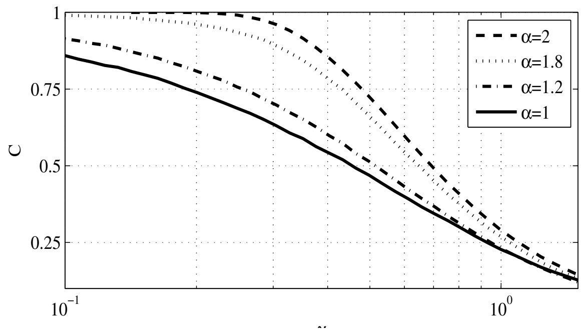

Fig. 1. Capacity of the channel with respect to α.

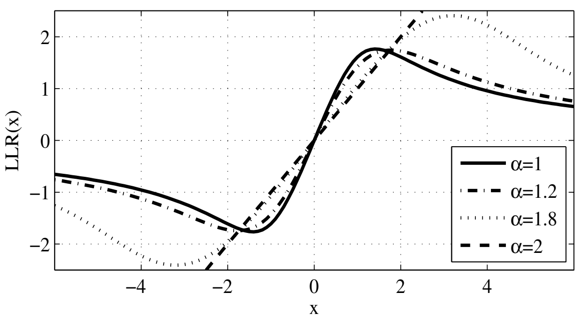

Fig. 2. LLR demapper.

are not necessary for our purpose which is simply to give some reference curves. The results are given in Fig. 1.

The capacity curves do a smooth transition between 0 and 1 bit per channel use according to γ. As expected, this transition spreads more when the impulsiveness increases. We can also notice that when the code rate, given by C, decreases the impact of α is weaker.

C. Channel-decoder mapping

To implement the BP decoder we first encode the channel outputs by their log-likelihood ratios (LLR),

$$\mathrm{LLR}(Y) = \log \frac{\Pr(Y|X=+1)}{\Pr(Y|X=-1)}. \tag{1}$$

The additive white Gaussian noise channel has the specific feature that the LLR expression has a closed and simple form: the LLR is proportional to the input. Thus, the input of the LDPC decoder is given by the received word rescaled. Unfortunately, this linear demapper is not suited to the more general case of AISN channels: when $\alpha < 2$, rescaling leads to poor performance [4].

The paper deals mainly with the transformation between the output of the channel and the input of the decoder. It is called in the rest of the paper *demapping*. The LLR expression (1) involves the pdf of the noise which is usually not accessible in closed-form. The demapper using LLR is thus only available through numerical approximation. Fig. 2 shows that the LLR demapper is highly non-linear when the noise is not Gaussian ($\alpha \neq 2$). Moreover, it depends on both channel parameters: the index α and the noise spread γ.

As expected, when the noise is impulsive, high value of the output should not be trusted; the LLR decreases. Beside, the span where the LLR *demapper* can be approximated with a linear function near 0 grows with the decrease of the impulsiveness of the noise. The limit case is the Gaussian case where the LLR is linear.

III. Clipping Optimization

To implement the *demapper*, using lookup tables or some approximations, would induce too much complexity. Moreover, the robustness and the efficiency of the channel parameters' estimation [10] has a significant impact on the *demapper* performance. The choice of another kind of *demapper* is then salutary. In this section, a simple clipping *demapper* is used and its parameters are optimized.

The clipping demapper scales the input by the "slope" P and clips values whose absolute value are greater than a threshold H, that is,

$$\mathrm{Clip}(y) = \begin{cases} P\,y & \text{if } -H < P\,y < +H, \\ H\ \mathrm{sign}(y) & \text{otherwise.} \end{cases} \tag{2}$$

Implementation of the clipping *demapper* is simple but the two parameters P and H have to be chosen in order to optimize the decoder performance. However, we cannot address this optimization problem in a direct manner due to the numerous parameters (α, γ, H, P) and the complexity of the LDPC decoding. To tackle the problem, we propose three different solutions: the LLR approximation, noise power reduction and an approach based on the density evolution.

We have two main concerns:

1) the efficiency of the decoding process;
2) can we find H and P values which do not depend on the channel parameters?

A. LLR function approximation

A first idea is to keep the clipping demapper close to the LLR. This can be done by choosing P as the slope of the tangent at the origin of the LLR. Indeed, we can see in Fig. 2 that a linear function is a good approximation of the LLR around 0. The clipping threshold H is then chosen as the maximum value of the LLR. Due to the symmetry of the channel, the slope is analytically expressed with the pdf of the noise, f_N, and its derivative f'_N,

$$P = \mathrm{LLR}'(0) = -2f'_N(1)/f_N(1). \tag{3}$$

To compute H and P, Fourier integrals of the characteristic function $\varphi_N(t)$ and of $it\varphi_N(t)$ are obtained through numerical integration, respectively for the pdf and its derivative.

The slope P depends on α as well as on γ. Fig. 3 shows two regimes: when $\gamma \lesssim 0.8$, the slope is almost independent of γ and when $\gamma \gtrsim 0.8$ the relation is of power-law kind. The P value at the frontier of both regimes is around 3.25. This aspect can be taken into account in order to reduce the size of lookup tables. The Gaussian case $\alpha = 2$ is still special because the optimal slope is inversely proportional to γ^2.

To obtain the threshold H, the maximum value of the LLR has to be numerically obtained by binary search algorithm. This parameter depends also on both α and γ. As expected, H decreases when γ increases or when α decreases.

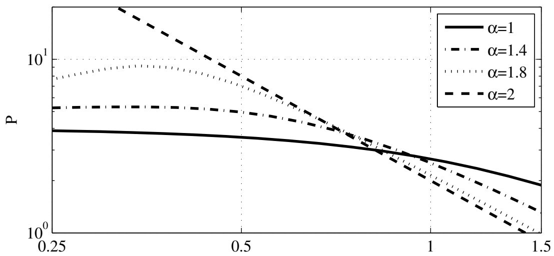

Fig. 3. Slope P wrt α and γ.

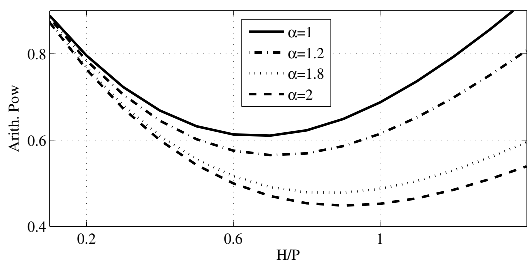

Fig. 4. H/P with respect to α for $\gamma = 0.7$.

B. Noise power minimization

The previous solution suffers from the dependency of H and P on the channel parameters α and γ. Especially, the noise spread γ may be difficult to estimate correctly and could be a fast changing parameter in a wireless communication networks as it involves the interfering neighbors [2]. To overcome this obstacle, we propose another solution. Except for the Gaussian case, the noise is impulsive and its power is infinite; however, the noise power after clipping, because of its finite support, is well defined and easy to estimate by standard methods. We propose to minimize it.

The noise estimation procedure is as follows: using a training sequence, the signal may be removed from the clipped channel output. We present in Fig. 4 the arithmetic power of the clipped noise as a function of the ratio H/P for different α. It shows that a specific value of H/P minimizes the clipped noise power. A similar behavior is observed whatever γ.

An optimal ratio H/P can be found but not optimal P and H. Consequently, once the optimal H/P is obtained, we still have to choose the slope P or the threshold H by another method. We propose to use the result from section III-A that links the slope to the LLR (see (3)). The relevance of this choice is that the slope depends mainly on α and much less on γ and α is a slow varying environmental parameter. To even avoid the dependency on α we propose to choose $P = 3.25$ which seems a reasonable value. In that way, this method does not necessitate any channel parameter estimation.

C. Density evolution

Asymptotically, BP decoding of infinite length LDPC undergoes a phase transition behavior: as for the Shannon's coding theorem, if the noise is too strong no correct decoding is possible; if however it is weak, the decoding must be perfect. Exponent α being fixed, the boundary of this two behaviors is given by a threshold θ on the value of γ. If $\gamma > \theta$ then the perfect decoding is not possible but it is if $\gamma < \theta$. The

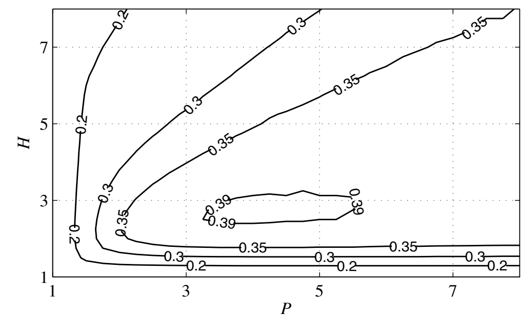

Fig. 5. Contour of the asymptotic performance of regular (3,6)-LDPC codes in AISN channel for $\alpha = 1.2$.

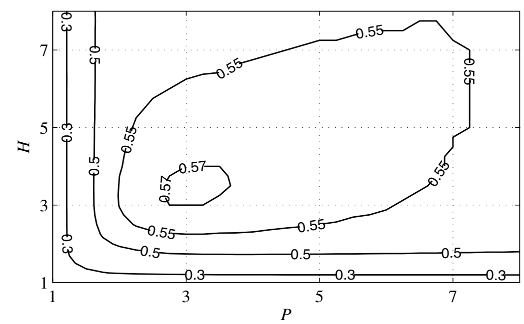

Fig. 6. Contour of the asymptotic performance of regular (3,6)-LDPC codes in AISN channel for $\alpha = 1.8$.

threshold θ depends on the LDPC code family. However, because of the finiteness of the codewords, the phase transition is not as sharp. But the threshold θ is nevertheless a good criterion for optimization.

This threshold is computable thanks to the density evolution (DE) method. Most of the time, this method permits to optimize the parameters of the LDPC code in order to approach the Shannon capacity of the channel [11]. In this paper, we do not use it to optimize the LDPC, supposed fixed, but we use it to optimize the parameters P and H. The main advantage is that, because of the existence of the threshold, the parameter γ does not influence the best clipping parameters' choice.

For each value of H and P, we obtain the threshold θ. Their optimal values are then computed by a maximization of $\theta(H, P)$. This choice asserts that if the noise level is below the threshold, then the BP decoder must decode perfectly an infinite length LDPC code. In Fig. 5, the contours of the thresholds found by the DE method when $\alpha = 1.2$ are drawn. The optimal H and P are $P \approx 4$ and $H \approx 3$. We observe a similar behavior on Fig. 6 for $\alpha = 1.8$. The chosen values are in this case $P \approx 3.2$ and $H \approx 4$.

Furthermore, a flat area around the optimal values can be observed. It means that a small error on H and P will not significantly degrade the decoding performance.

Table I records the optimal values of H and P for different channel parameters α. Interestingly, we can observe that when the noise is sufficiently impulsive, $\alpha \lesssim 1.7$, the clipping level is below the signal level.

TABLE I
CLIPPING DEMAPPER PARAMETERS ACCORDING TO α

α	1.0	1.2	1.4	1.6	1.8	2.0
H	≈ 2.8	≈ 3	≈ 3	≈ 3.2	≈ 3.5	∞
P	≈ 4	≈ 4	≈ 4	≈ 3.5	≈ 3.2	≈ 2.5

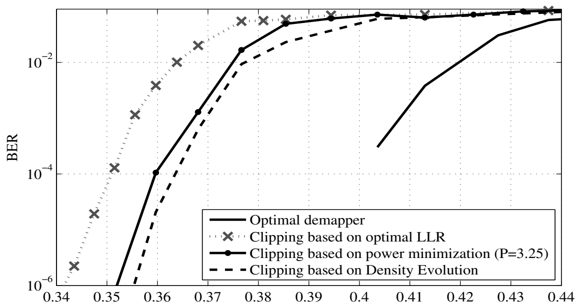

Fig. 7. Comparison of the methods on channel with $\alpha = 1.2$.

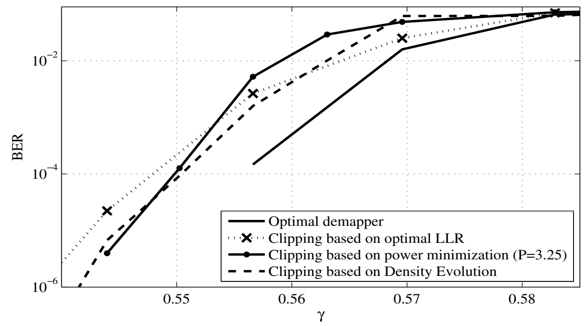

Fig. 8. Comparison of the methods on channel with $\alpha = 1.8$.

IV. SIMULATIONS

In this section, we compare in terms of bit error rate (BER) the clipping demappers obtained by the three methods previously presented. The code is a $(3, 6)$-regular LDPC code of length 20000 bits. The channels are AISN channels with $\alpha = 1.2$ (Fig. 7) and $\alpha = 1.8$ (Fig. 8). These values were chosen because the results obtained are representative of the behaviors under weak and strong impulsiveness respectively. Note however, that the comparison of the results for different values of alpha is not straightforward as the notion of SNR is not well-defined [8].

The different solutions give close results. A slight advantage can be observed for the approach based on DE. Moreover, this approach is highly interesting because H and P only depends on α and not on the dispersion. Besides, the dependence on α of the clipping parameters is rather weak, as seen in table I, except when α tends towards 2. Consequently, this method can handle a reasonable estimation error on α.

The second approach, based on the arithmetic power of the clipped noise, is also robust because, the noise power, and consequently the optimal value of H/P, is simple to estimate thanks to a training sequence. An efficient demapper is then computed taking $P = 3.25$.

The last solution gives the poorest performance, especially when impulsiveness increases. We have noticed that this is essentially due to the choice of the threshold H. Besides, H and P depend on the two channel parameters α and γ and look-up tables will also be necessary due to the complexity of the numerical evaluation of the parameters.

V. CONCLUSION

A careful design of the LLR calculation at the input of the BP decoder of LDPC is necessary when noise is impulsive. Based on a clipping approach, we propose three solutions to obtain the two parameters (slope P and threshold H) of the demapper: inspired by the optimal LLR, based on the clipped noise power and using Density Evolution. If all three solutions give relatively similar results, we prefer the following approach: (a) if we can estimate α, a look-up table defined thanks to the DE gives good performance. A further study on the degradation of the performance when an estimation error is done is necessary; (b) we can also link the power of the clipped noise estimated thanks to a training sequence to the value of H/P. Then, using a slope $P = 3.25$, the obtained performance are very satisfying.

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