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Optimal Capacity Relay Node Placement in a Multi-hop Network on a Line

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***Abstract*—We use information theoretic achievable rate formulas for the multi-relay channel to study the problem of optimal placement of relay nodes along the straight line joining a source node and a destination node. The achievable rate formulas that we utilize are for full-duplex radios at the relays and decode-and-forward relaying. For the single relay case, and individual power constraints at the source node and the relay node, we provide explicit formulas for the optimal relay location and the optimal power allocation to the source-relay channel, for the exponential and the power-law path-loss channel models. For the multiple relay case, we consider exponential path-loss and a total power constraint over the source and the relays, and derive an optimization problem, the solution of which provides the optimal relay locations. Numerical results suggest that at low attenuation the relays are mostly clustered close to the source in order to be able to cooperate among themselves, whereas at high attenuation they are uniformly placed and work as repeaters. We also prove that a constant rate independent of the attenuation in the network can be achieved by placing a large enough number of relay nodes uniformly between the source and the destination, under the exponential path-loss model with total power constraint.**

I. Introduction

In this paper we consider the problem of maximizing the data rate between a source node (e.g., a wireless sensor) and a destination (e.g., a sink node in a wireless sensor network) by means of optimally placing relay nodes on the line segment joining the source and the destination; see Figure 1. In order to understand the fundamental trade-offs involved in such a problem, we consider an information theoretic model. For a placement of the relay nodes along the line and allocation of transmission powers to these relays, we model the "quality" of communication between the source and the destination by the information theoretic achievable rate of the relay channel[1]. The relays are equipped with full-duplex radios[2], and carry out decode-and-forward relaying. We consider scalar, memoryless, time-invariant, additive white Gaussian noise (AWGN) channels. A path-loss model is important for our study, and we consider both power-law and exponential path-loss models.

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[1]The term "relay channel" will, in this paper, include the term "multi-relay channel."

[2]See [1] for recent efforts to realize practical full-duplex radios.

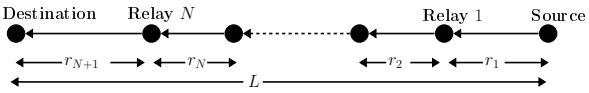


Fig. 1. A source and a destination connected by a multi-hop path comprising N relay nodes along a line.

A. Related Work

A formulation of the problem of relay placement requires a model of the wireless network at the physical (PHY) and medium access control (MAC) layers. Most researchers have adopted the link scheduling and interference model, i.e., a scheduling algorithm determines radio resource allocation (channel and power) and interference is treated as noise (see [2]). But node placement for throughput maximization with this model seems to be intractable because the optimal throughput is obtained by first solving for the optimum schedule assuming fixed node locations, followed by an optimization over those locations. Hence, with such a model, there appears to be little work on the problem of jointly optimizing the relay node placement and the transmission schedule. In [3], the authors considered placing a set of nodes in an existing network such that certain network utility (e.g., total transmit power) is optimized subject to a set of linear constraints on link rates. They posed the problem as one of geometric programming assuming exponential path-loss, and showed that it can be solved in a distributed fashion. To the best of our knowledge, there appears to be no other work which considers joint optimization of link scheduling and node placement using link scheduling model.

On the other hand, an information theoretic model for a wireless network often provides a closed-form expression for the channel capacity, or at least an achievable rate region. These results are asymptotic, and make idealized assumptions such as full-duplex radios, perfect interference cancellation, etc., but provide algebraic expressions that can be used to formulate tractable optimization problems. The results from these formulations can provide useful insights. In the context of optimal relay placement, some researchers have already exploited this approach. For example, Thakur et al. in [4] report on the problem of placing a single relay node to maximize the capacity of a broadcast relay channel in a wideband regime. The linear deterministic channel model ([5])

is used in [6] to study the problem of placing two or more relay nodes along a line so as to maximize the end-to-end data rate. Our present paper is in a similar spirit; however, we use the achievable rate formulas for the N-relay channel (with decode and forward relays) to study the problem of placing relays on a line under individual node power constraints as well as with sum power constraints over the source and the relays.

B. Our Contribution

- In Section III, we consider the problem of placing a single relay with individual power constraints at the source and the relay. In this context, we provide explicit formulas for the optimal relay location and the optimal source power split (between providing new information to the relay and cooperating with the relay to assist the destination), for the exponential path-loss model (Theorem 2) and for the power law path-loss model (Theorem 3). We find that at low attenuation it is better to place the relay near the source, whereas at very high attenuation the relay should be placed at half-distance between the source and the destination.
- In Section IV, we focus on the N relay placement problem with exponential path-loss model and a sum power constraint among the source and the relays. For given relay locations, the optimal power split among the nodes and the achievable rate are given in Theorem 4 in terms of the channel gains. We explicitly solve the single relay placement problem in this context (Theorem 5). A numerical study shows that, the relay nodes are clustered near the source at low attenuation and are placed uniformly between the source and the destination at high attenuation. We have also studied the asymptotic behaviour of the achievable rate R_N when N relay nodes are placed uniformly on a line of fixed length, and show that for a total power constraint P_T among the the source and the relays, $\liminf_{N\to\infty} R_N \geq C(\frac{P_T}{2\sigma^2})$, where $C(\cdot)$ is the AWGN capacity formula and σ^2 is the power of the additive white Gaussian noise at each node.

The rest of the paper is organized as follows. In Section II, we describe our system model and notation. In Section III, node placement with per-node power constraint has been discussed. Node placement for total power constraint has been discussed in Section IV. Conclusions have been drawn in Section V.

II. System Model and Notation

The multi-relay channel was studied in [7] and [8] and is an extension of the single relay model presented in [9]. We consider a network deployed on a line with a source node, a destination node at the end of the line, and N full-duplex relay nodes as shown in Figure 1. The relay nodes are numbered as $1, 2, \cdots, N$. The source and destination are indexed by 0 and $N+1$, respectively. The distance of the k-th node from the source is denoted by $y_k := r_1 + r_2 + \cdots + r_k$. Thus, $y_{N+1} = L$. As in [7] and [8], we consider the scalar, time-invariant, memoryless, additive white Gaussian noise setting. A symbol transmitted by node i is received at node j after multiplication by the (positive, real valued) channel gain $h_{i,j}$. The Gaussian additive noise at any receiver is independent and identically distributed from symbol to symbol and has variance σ^2. The *power gain* from Node i to Node j is denoted by $g_{i,j} = h_{i,j}^2$. We model the power gain via two alternative path-loss models: exponential path-loss and power-law path-loss. The power gain at a distance r is $e^{-\rho r}$ for the exponential path-loss model and $r^{-\eta}$ for the power-law path-loss model, where $\rho > 0$, $\eta > 1$. These path- loss models have their roots in the physics of wireless channels ([10]). For the exponential path-loss model, we will denote $\lambda := \rho L$. Under the exponential path-loss model, the channel gains and power gains in the line network become multiplicative, e.g., $h_{i,i+2} = h_{i,i+1}h_{i+1,i+2}$ and $g_{i,i+2} = g_{i,i+1}g_{i+1,i+2}$ for $i \in \{0, 1, \cdots, N-1\}$. In this case, we define $g_{i,i} = 1$ and $h_{i,i} = 1$. The power-law path-loss expression fails to characterize near-field transmission, since it goes to ∞ as $r \to 0$. One alternative is the "modified power-law path-loss" model where the path-loss is $\min\{r^{-\eta}, b^{-\eta}\}$ with $b > 0$ a reference distance. In this paper we consider both power-law and modified power-law path loss models, apart from the exponential path-loss model.

A. The Multi-Relay Channel

For the multi-relay channel, we denote the symbol transmitted by the i-th node at time t (t is discrete) by $X_i(t)$ for $i = 0, 1, \cdots, N$. $Z_k(t) \sim \mathcal{N}(0, \sigma^2)$ is the additive white Gaussian noise at node k and time t, and is assumed to be independent and identically distributed across k and t. Thus, at symbol time t, node $k, 1 \leq k \leq N+1$ receives:

$$Y_k(t) = \sum_{j\in\{0,1,\cdots,N\}, j\neq k} h_{j,k}X_j(t) + Z_k(t)$$

In [7], the authors showed that an inner bound to the capacity of this network is given by (defining $C(x) := \frac{1}{2}\log_2(1+x)$):

$$R = \min_{1\leq k\leq N+1} C\left(\frac{1}{\sigma^2}\sum_{j=1}^{k}\Big(\sum_{i=0}^{j-1} h_{i,k}\sqrt{P_{i,j}}\Big)^2\right) \qquad (1)$$

where $P_{i,j}$ denotes the power at which node i transmits to node j. Its significance will be clear later in this section.

In order to provide insight into the expression in Equation (1) and the relay placement results in this paper, we provide a descriptive overview of the coding and decoding scheme described in [7]. Transmissions take place via block codes of T symbols each. The transmission blocks at the source and the N relays are synchronized. The coding and decoding scheme is such that a message generated at the source at the beginning of block $b, b \geq 1$, is decoded by the destination at the end of block $b+N$, i.e., $N+1$ block durations after the message was generated (with probability tending to 1, as $T \to \infty$). Thus, at the end of B blocks, $B \geq N+1$, the destination is able to decode $B-N$ messages. It follows, by taking $B \to \infty$, that, if the code rate is R bits per symbol, then an information rate

of R bits per symbol can be achieved from the source to the destination.

As mentioned earlier, we index the source by 0, the relays by $k, 1 \leq k \leq N$, and the destination by $N+1$. There are $(N+1)^2$ independent Gaussian random codebooks, each containing 2^{TR} codes, each code being of length T; these codebooks are available to all nodes. At the beginning of block b, the source generates a new message w_b, and, in this description, we assume that each node $k, 1 \leq k \leq N+1$, has a reliable estimate of all the messages $w_{b-j}, j \geq k$. In block b, the source uses a new codebook to encode w_b. In addition, relay $k, 1 \leq k \leq N$, and *all* the transmitters previous to it (indexed $0 \leq j \leq k-1$), use *another* codebook to encode w_{b-k} (or their estimate of it). Thus, if the relays $1, 2, \cdots, k$ have a perfect estimate of w_{b-k} at the beginning of block b, they will transmit the same codeword for w_{b-k}. Therefore, in block b, the source and relays $1, 2, \cdots, k$ *coherently transmit* the codeword for w_{b-k}. In this manner, in block b, transmitter $k, 0 \leq k \leq N$, generates $N+1-k$ codewords, corresponding to $w_{b-k}, w_{b-k-1}, \cdots, w_{b-N}$, which are transmitted with powers $P_{k,k+1}, P_{k,k+2}, \cdots, P_{k,N+1}$. In block b, node $k, 1 \leq k \leq N+1$, receives a superposition of transmissions from all other nodes. Assuming that node k knows all the powers, and all the channel gains, and recalling that it has a reliable estimate of all the messages $w_{b-j}, j \geq k$, it can subtract the interference from transmitters $k+1, k+2, \cdots, N$. At the end of block b, after subtracting the signals it knows, node k is left with the k received signals from nodes $0, 1, \cdots, (k-1)$ (received in blocks $b, b-1, \cdots, b-k+1$), which all carry an encoding of the message w_{b-k+1}. These k signals are then jointly used to decode w_{b-k+1}, using joint typicality decoding. The codebooks are cycled through in a manner so that in any block all nodes encoding a message (or their estimate of it) use the same codebook, but different (thus, independent) codebooks are used for different messages. Under this encoding and decoding scheme, relatively simple arguments lead to the conclusion that any rate strictly less than R displayed in Equation (1) is achievable.

From the above description we see that a node receives information about a message in two ways (i) by the message being directed to it cooperatively by all the previous nodes, and (ii) by overhearing previous transmissions of the message to the previous nodes. The higher the channel attenuation, the less will be the contribution of farther nodes, "overheard" transmissions become less relevant, and coherent transmission reduces to a simple transmission from the previous relay. The system is then closer to simple store-and-forward relaying.

The authors of [7] have shown that any rate strictly less than R is achievable through this coding and decoding scheme which involves coherent multi-stage relaying and interference subtraction. This achievable rate formula can also be obtained from the capacity formula of a physically degraded multi-relay channel (see [8] for the channel model), since the capacity of the degraded relay channel is a lower bound to the actual channel capacity. In this paper, we will seek to optimize R over power allocations to the nodes and the node locations. Also, $\log(\cdot)$ in this paper will mean the natural logarithm unless the base is specified. We denote the value of R optimized over power allocation and relay locations by R^*.

For the single relay channel, $N = 1$. Thus, by (1), an achievable rate is given by (see also [9]):

$$R = \min \quad \left\{ \begin{array}{l} C\left(\frac{g_{0,1}P_{0,1}}{\sigma^2}\right), \\ C\left(\frac{g_{0,2}P_{0,1} + (h_{0,2}\sqrt{P_{0,2}} + h_{1,2}\sqrt{P_{1,2}})^2}{\sigma^2}\right) \end{array} \right\} \quad (2)$$

The following result justifies our aim to seek optimal relay placement on the line joining the source and the destination (rather than anywhere else on the plane).

Theorem 1: For given source and destination locations, the relays should always be placed on the line segment joining the source and the destination in order to maximize the end-to-end data rate between the source and the destination.[3]

III. Single Relay Node Placement: Node Power Constraints

In this section, we aim at placing a single relay node between the source and the destination in order to maximize the achievable rate R given by Equation (2). Let the distance between the source and the relay be r, i.e., $r_1 = r$. Let $\alpha := \frac{P_{0,1}}{P_0}$. We assume that the source and the relay use the same transmit power P. Hence, $P_{0,1} = \alpha P$, $P_{0,2} = (1-\alpha)P$ and $P_{1,2} = P$. Thus, for a given placement of the relay node, we obtain:

$$R = \min \quad \left\{ \begin{array}{l} C\left(\frac{\alpha g_{0,1} P}{\sigma^2}\right), \\ C\left(\frac{P}{\sigma^2}\left(g_{0,2} + g_{1,2} + 2\sqrt{(1-\alpha)g_{0,2}g_{1,2}}\right)\right) \end{array} \right\} \quad (5)$$

Maximizing over α and r, and exploiting the monotonicity of $C(\cdot)$ yields the following problem:

$$\max_{r\in[0,L]} \max_{\alpha\in[0,1]} \min\left\{\alpha g_{0,1},\, g_{0,2} + g_{1,2} + 2\sqrt{(1-\alpha)g_{0,2}g_{1,2}}\right\} \quad (6)$$

Note that $g_{0,1}$ and $g_{1,2}$ in the above equation depend on r, but $g_{0,2}$ does not depend on r.

A. Exponential Path Loss Model

1) Optimum Relay Location: Here $g_{0,1} = e^{-\rho r}$, $g_{1,2} = e^{-\rho(L-r)}$ and $g_{0,2} = e^{-\rho L}$. Let $\lambda := \rho L$, and the optimum relay location be r^*. Let $x^* := \frac{r^*}{L}$ be the normalized optimal relay location. Let α^* be the optimum value of α when relay is optimally placed.

Theorem 2: For $N = 1$, under the exponential path-loss model, if the source and relay have same power constraint P, then there is a unique optimum relay location r^* and the following hold:

(i) $x^* = \max\{x_+, 0\}$ where $x_+ := -\frac{1}{\lambda}\log\left(2e^{-\lambda} + e^{-\frac{\lambda}{2}}\right)$.
(ii) For $0 \leq \lambda \leq \log 2$, $x^* = 0$, $\alpha^* = 1$ and $R^* = C(\frac{P}{\sigma^2})$.

[3]Detailed proofs of all the theorems in this paper can be found in [11]

$$\alpha^* = \frac{e^{-\lambda}}{\left(2e^{-\lambda}+e^{-\frac{\lambda}{2}}\right)}\left(\sqrt{1-\frac{e^{-\lambda}}{(2e^{-\lambda}+e^{-\frac{\lambda}{2}})^2}}+\sqrt{\frac{1}{(2e^{-\lambda}+e^{-\frac{\lambda}{2}})}\left(1-\frac{e^{-\lambda}}{(2e^{-\lambda}+e^{-\frac{\lambda}{2}})}\right)}\right)^2 \quad (3)$$

$$R^* = C\left(\frac{P}{\sigma^2}e^{-\lambda}\left(\sqrt{1-\frac{e^{-\lambda}}{(2e^{-\lambda}+e^{-\frac{\lambda}{2}})^2}}+\sqrt{\frac{1}{(2e^{-\lambda}+e^{-\frac{\lambda}{2}})}\left(1-\frac{e^{-\lambda}}{(2e^{-\lambda}+e^{-\frac{\lambda}{2}})}\right)}\right)^2\right) \quad (4)$$

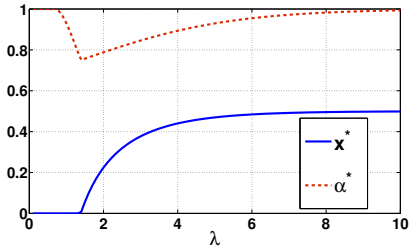


Fig. 2. Single Relay, exponential path-loss, node power constraint: x^* and α^* versus λ.

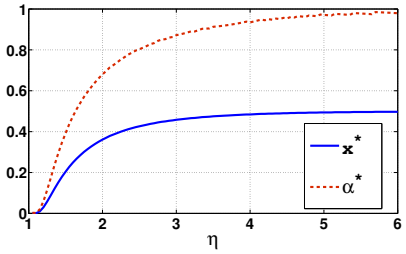


Fig. 3. Power law path-loss, single relay node, individual power constraint: x^* and α^* versus η.

(iii) For $\log 2 \leq \lambda \leq \log 4$, $x^* = 0$, $\alpha^* = 4e^{-\lambda}(1-e^{-\lambda})$ and $R^* = C(\frac{P}{\sigma^2}4e^{-\lambda}(1-e^{-\lambda}))$.

(iv) For $\lambda \geq \log 4$, $x^* = x_+$. Moreover, α^* and R^* are given by Equations (3) and (4).

2) Numerical Work: In Figure 2, we plot x^* and α^* provided by Theorem 2, versus λ. Recalling the discussion of the coding/decoding scheme in Section II, we note that the relay provides two benefits to the source: (i) being nearer to the source than the destination, the power required to transmit a message to the relay is less than that to the destination, and (ii) having received the message, the relay coherently assists the source in resolving ambiguity at the destination. These two benefits correspond to the two terms in Equation (2). Hence, Theorem 2 and Figure 2 provide the following insights:

- At very low attenuation ($\lambda \leq \log 2$), $x^* = 0$ (since $x_+ \leq 0$). Since $g_{1,2} \geq \frac{1}{2}$ and $g_{0,2} \geq \frac{1}{2}$, we will always have $g_{0,2}+g_{1,2}+2\sqrt{(1-\alpha)g_{0,2}g_{1,2}} \geq \alpha g_{0,1}$ for all $\alpha \in [0,1]$ and for all $r \in [0, L]$. The minimum of the two terms in Equation (6) is maximized by $\alpha = 1$ and $r = 0$.
- For $\log 2 \leq \lambda \leq \log 4$, we again have $x^* \leq 0$. α^* decreases from 1 to $\frac{3}{4}$ as λ increases. In this case since attenuation is higher, the source needs to direct some power to the destination for transmitting coherently with the relay to the destination to balance the source to relay data rate. $\lambda = \log 4$ is the critical value of λ at which source to relay channel ceases to be the bottleneck. Hence, for $\lambda \in [\log 2, \log 4]$, we still place the relay at the source but the source reserves some of its power to transmit to the destination coherently with the relay's transmission to the destination. As attenuation increases, the source has to direct more power to the destination, and so α^* decreases with λ.
- For $\lambda \geq \log 4$, $x^* \geq 0$. Since attenuation is high, it is no longer optimal to keep the relay close to the source. Thus, x^* increases with λ. As $\lambda \to \infty$, $x^* \to \frac{1}{2}$ and $\alpha^* \to 1$. We observe that the ratio of powers received by the destination from the source and the relay is less than $e^{-\frac{\lambda}{2}}$, since $x^* \leq \frac{1}{2}$. This ratio tends to zero as $\lambda \to \infty$. Thus, at high attenuation, the source transmits at full power to the relay and the relay acts just as a repeater.
- We observe that $x^* \leq \frac{1}{2}$. The two data rates in Equation (5) can be equal only if $r \leq \frac{L}{2}$. Otherwise we will readily have $g_{0,1} \leq g_{0,2} + g_{1,2}$, which means that the two rates will not be equal.

B. Power Law Path Loss Model

1) Optimum Relay Location: For the power-law path-loss model, $g_{0,1} = r^{-\eta}$, $g_{0,2} = L^{-\eta}$ and $g_{1,2} = (L-r)^{-\eta}$. Let x^* be the normalized optimum relay location which maximizes R. Then the following theorem states how to compute x^*.

Theorem 3: (i) For the single relay channel and power-law path-loss model (with $\eta > 1$), there is a unique placement point for the relay on the line joining the source and the destination, maximizing R. The normalized distance of the point from source node is $x^* < \frac{1}{2}$, where x^* is precisely the unique real root p of the Equation $(x^{-\eta+1}-1)^2(1-(\frac{1}{x}-1)^{-\eta}) = (1-x)^{-\eta} - (\frac{1}{x}-1)^{-\eta}$ in the interval $(0, \frac{1}{2}]$.

(ii) For the "modified power-law path-loss" model with $2b < L$, the normalized optimum relay location is $x^* = \max\{p, \frac{b}{L}\}$.

2) Numerical Work: The variation of x^* and α^* as a function of η for the power-law path loss model are shown in Figure 3. As η increases, both x^* and α^* increase. For large η, they are close to 0.5 and 1 respectively, which means that the relay works just as a repeater. At low attenuation the behaviour is different from exponential path-loss because in that case the two rates in Equation (5) can be equalized since channels gains are unbounded. For small η, relay is placed close to the source, $g_{0,1}$ is high and hence a small α suffices to equalize the two rates. Thus, we are in a situation similar to the $\lambda \geq \log 4$ case for exponential path-loss model. Hence, α^* and x^* increase with λ.

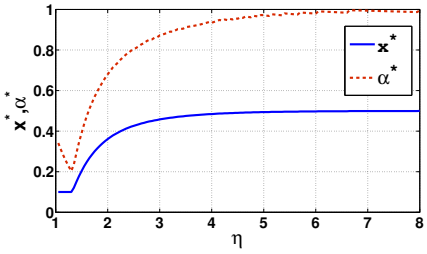


Fig. 4. Modified power-law path-loss (with $\frac{b}{L} = 0.1$), single relay node, individual power constraint: x^* and α^* versus η.

On the other hand, the variation of x^* and α^* with η assuming the modified power-law path-loss model are shown in Figure 4, with $\frac{b}{L} = 0.1$. Here, for small values of η, $p \leq 0.1$ and hence $x^* = 0.1$. Beyond the point where 0.1 is the solution of $(x^{-\eta+1}-1)^2(1-(\frac{1}{x}-1)^{-\eta}) = (1-x)^{-\eta} - (\frac{1}{x}-1)^{-\eta}$, the behaviour is similar to the power-law path-loss model. However, for those values of η which result in $p \leq \frac{b}{L}$, the value of α^* decreases with η. This happens because in this region, if η increases, $g_{0,1}$ remains fixed but $g_{0,2}$ decreases.

IV. Multiple Relay Placement : Sum Power Constraint

In this section, we derive the optimal placement of relay nodes to maximize R (see Equation (1)), subject to a total power constraint on the source and relay nodes given by $\sum_{i=0}^{N} P_i = P_T$. We consider only the exponential path-loss model. We will first maximize R in Equation (1) over $P_{i,j}, 0 \leq i < j \leq (N+1)$ for any given placement of nodes (i.e., given $y_1, y_2, \cdots, y_N$). This will provide an expression of achievable rate in terms of channel gains, which has to be maximized over $y_1, y_2, \cdots, y_N$. Let $\gamma_k := \sum_{i=0}^{k-1} P_{i,k}$ for $k \in \{1, 2, \cdots, N+1\}$. Hence, the sum power constraint becomes $\sum_{k=1}^{N+1} \gamma_k = P_T$.

Theorem 4: (i) For fixed location of relay nodes, the optimal power allocation that maximizes the achievable rate for the sum power constraint is given by:

$$P_{i,j} = \begin{cases} \frac{g_{i,j}}{\sum_{l=0}^{j-1} g_{l,j}} \gamma_j & \forall 0 \leq i < j \leq (N+1) \\ 0, & \text{if } j \leq i \end{cases} \quad (7)$$

where

$$\gamma_1 = \frac{P_T}{1 + g_{0,1} \sum_{k=2}^{N+1} \frac{(g_{0,k-1} - g_{0,k})}{g_{0,k} g_{0,k-1} \sum_{l=0}^{k-1} \frac{1}{g_{0,l}}}} \quad (8)$$

$$\gamma_j = \frac{g_{0,1} \frac{(g_{0,j-1} - g_{0,j})}{g_{0,j} g_{0,j-1} \sum_{l=0}^{j-1} \frac{1}{g_{0,l}}}}{1 + g_{0,1} \sum_{k=2}^{N+1} \frac{(g_{0,k-1} - g_{0,k})}{g_{0,k} g_{0,k-1} \sum_{l=0}^{k-1} \frac{1}{g_{0,l}}}} P_T \quad \forall j \geq 2$$

(ii) The achievable rate optimized over the power allocation for a given placement of nodes is given by:

$$R_{P_T}^{opt}(y_1, y_2, \cdots, y_N) = C\left(\frac{\frac{P_T}{\sigma^2}}{\frac{1}{g_{0,1}} + \sum_{k=2}^{N+1} \frac{(g_{0,k-1} - g_{0,k})}{g_{0,k} g_{0,k-1} \sum_{l=0}^{k-1} \frac{1}{g_{0,l}}}} \right) \quad (9)$$

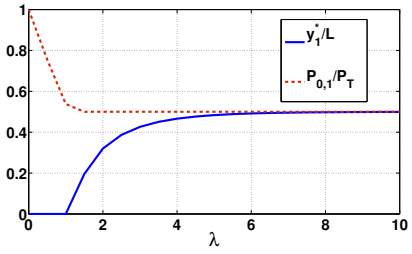


Fig. 6. Single relay placement, total power constraint, exponential path-loss: $\frac{y_1^*}{L}$ and optimum $\frac{P_{0,1}}{P_T}$ versus λ.

Proof: See Appendix A. ■

Remarks and Discussion:

- We find that in order to maximize $R_{P_T}^{opt}(y_1, y_2, \cdots, y_N)$, we need to place the relay nodes such that $\frac{1}{g_{0,1}} + \sum_{k=2}^{N+1} \frac{(g_{0,k-1} - g_{0,k})}{g_{0,k} g_{0,k-1} \sum_{l=0}^{k-1} \frac{1}{g_{0,l}}}$ is minimized.
- From Equation (7), it follows that any node $k < j$ will transmit at a higher power to node j, compared to any node preceding node k.
- Note that we have derived Theorem 4 using the fact that $g_{0,k}$ is nonincreasing in k. If there exists some $k \geq 1$ such that $g_{0,k} = g_{0,k+1}$, i.e, if k-th and $(k+1)$-st nodes are placed at the same position, then $\gamma_{k+1} = 0$, i.e., the nodes $i < k$ do not direct any power specifically to relay $k+1$. However, relay $k+1$ can decode the symbols received at relay k, and those transmitted by relay k. Then relay $(k+1)$ can transmit coherently with the nodes $l \leq k$ to improve effective received power in the nodes $j > k+1$.

A. Optimal Placement of a Single Relay Node

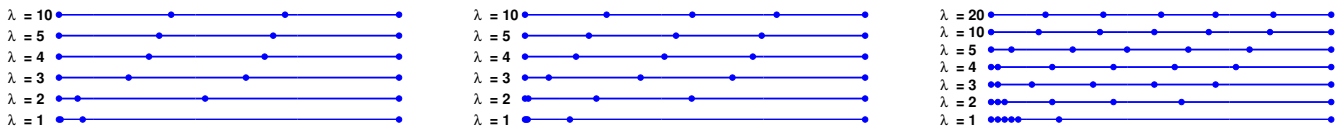
Theorem 5: For the single relay node placement problem with sum power constraint and exponential path-loss model, the normalized optimum relay location $\frac{y_1^*}{L}$, power allocation and optimized achievable rate are given as follows :

(i) *For* $\lambda \leq \log 3$, $\frac{y_1^*}{L} = 0$, $P_{0,1} = \frac{2P_T}{e^\lambda + 1}$, $P_{0,2} = P_{1,2} = \frac{e^\lambda - 1}{e^\lambda + 1} \frac{P_T}{2}$ and $R^* = C\left(\frac{2P_T}{(e^\lambda+1)\sigma^2}\right)$.

(ii) *For* $\lambda \geq \log 3$, $\frac{y_1^*}{L} = \frac{1}{\lambda} \log\left(\sqrt{e^\lambda + 1} - 1\right)$, $P_{0,1} = \frac{P_T}{2}$, $P_{0,2} = \frac{1}{\sqrt{e^\lambda+1}} \frac{P_T}{2}$, $P_{1,2} = \frac{\sqrt{e^\lambda+1}-1}{\sqrt{e^\lambda+1}} \frac{P_T}{2}$ and $R^* = C\left(\frac{1}{\sqrt{e^\lambda+1}-1} \frac{P_T}{2\sigma^2}\right)$

Remarks and Discussion:

- It is easy to check that R^* obtained in Theorem 5 is strictly greater than the AWGN capacity $C\left(\frac{P_T}{\sigma^2} e^{-\lambda}\right)$ for all $\lambda > 0$. This happens because the source and relay transmit coherently to the destination. R^* becomes equal to the AWGN capacity only at $\lambda = 0$. At $\lambda = 0$, we do not use the relay since the destination can decode any message that the relay is able to decode.
- The variation of $\frac{y_1^*}{L}$ and $\frac{P_{0,1}}{P_T}$ with λ has been shown in Figure 6. We observe that $\lim_{\lambda\to\infty} \frac{y_1^*}{L} = \frac{1}{2}$, $\lim_{\lambda\to\infty} P_{0,2} = 0$ and $\lim_{\lambda\to 0} P_{0,1} = P_T$. For large



(a) $N = 2$ (b) $N = 3$ (c) $N = 5$

Fig. 5. N relays, exponential path-loss model: depiction of the optimal relay positions for $N = 2, 3, 5$ for various values of λ.

values of λ, source and relay cooperation provides negligible benefit since source to destination attenuation is very high. So it is optimal to place the relay at a distance $\frac{L}{2}$. The relay works as a repeater which forwards data received from the source to the destination.

B. Optimal Relay Placement for a Multi-Relay Channel

As we discussed earlier, we need to place N relay nodes such that $\frac{1}{g_{0,1}} + \sum_{k=2}^{N+1} \frac{(g_{0,k-1} - g_{0,k})}{g_{0,k} g_{0,k-1} \sum_{l=0}^{k-1} \frac{1}{g_{0,l}}}$ is minimized. Here $g_{0,k} = e^{-\rho y_k}$. We have the constraint $0 \leq y_1 \leq y_2 \leq \cdots \leq y_N \leq y_{N+1} = L$. Now, writing $z_k = e^{\rho y_k}$, and defining $z_0 := 1$, we arrive at the following optimization problem:

$$\min \left\{ z_1 + \sum_{k=2}^{N+1} \frac{z_k - z_{k-1}}{\sum_{l=0}^{k-1} z_l} \right\}$$
$$s.t \quad 1 \leq z_1 \leq \cdots \leq z_N \leq z_{N+1} = e^{\rho L} \qquad (10)$$

One special feature of the objective function is that it is convex in each of the variables $z_1, z_2, \cdots, z_N$. The objective function is sum of linear fractionals, and constraints are linear.

Remark: From optimization problem (10) we observe that optimum $z_1, z_2, \cdots, z_N$ depend only on $\lambda := \rho L$. Since $z_k = e^{\lambda \frac{y_k}{L}}$, we find that normalized optimal distance of relays from the source depend only on λ.

Theorem 6: For fixed ρ, L and σ^2, the optimized achievable rate R^* for a sum power constraint *strictly* increases with the number of relay nodes.

C. Numerical Work

We discretize the interval $[0, L]$ and run a search program to find normalized optimal relay locations for different values of λ and N. The result is summarized in Figure 5. We observe that at low attenuation (small λ), relay nodes are clustered near the source node and are often at the source node, whereas at high attenuation (large λ) they are almost uniformly placed along the line. For large λ, the effect of long distance between any two adjacent nodes dominates the gain obtained by coherent relaying. Hence, it is beneficial to minimize the maximum distance between any two adjacent nodes and thus multihopping is a better strategy in this case. On the other hand, if attenuation is low, the gain obtained by coherent transmission is dominant. In order to allow this, relays should be able to receive sufficient information from their previous nodes. Thus, they tend to be clustered near the source.[4]

[4]At low attenuation, one or more than one relay nodes are placed very close to the source. We believe that some of them will indeed be placed at the source, but it is not showing up in Figure 5 because we discretized the interval $[0, L]$ to run a search program to find optimum relay locations.

D. Uniformly Placed Relays, Large N

If the relays are uniformly placed, the behaviour of $R_{P_T}^{opt}(y_1, y_2, \cdots, y_N)$ (called R_N in the next theorem) for large number of relays is captured by the following Theorem.

Theorem 7: For the exponential path-loss model with total power constraint, if N relay nodes are placed uniformly between the source and destination, resulting in R_N achievable rate, then $\liminf_N R_N \geq C\left(\frac{P_T/\sigma^2}{2-e^{-\lambda}}\right)$ and $\limsup_N R_N \leq C\left(\frac{P_T}{\sigma^2}\right)$.

Proof: See Appendix A. ■

Remark: From Theorem 7, it is clear that we can ensure at least $C(\frac{P_T}{2\sigma^2})$ data rate (roughly) by placing a large enough number of relay nodes.

V. Conclusion

We have studied the problem of placing relay nodes in a line network in order to maximize the end-to-end achievable data rate. We obtained explicit formulas for the optimal location of a single relay node with individual power constraints on the source and the relay under power-law and exponential path-loss models. For the multi-relay channel with exponential path-loss and total power constraint, we derived an expression for the achievable rate and formulated the optimization problem to maximize the end-to-end data rate. We solved explicitly the single relay placement problem with exponential path-loss and total power constraint. The asymptotic performance of uniform relay placement was also studied for the total power constraint. However, the communication model assumed in this paper has certain impractical aspects: (i) all the nodes should have global knowledge of the codebooks, channel gains and transmit powers, (ii) relay nodes need to be full-duplex, (iii) global synchronization at block level is required. In our future work, we will consider relay placement problems in which one or more of these assumptions is relaxed.

Appendix A
Multiple Relay Placement : Sum Power Constraint

A. Proof of Theorem 4

We want to maximize R given in Equation (1) subject to the total power constraint, assuming fixed relay locations. Let us consider $C\left(\frac{1}{\sigma^2}\sum_{j=1}^{k}(\sum_{i=0}^{j-1} h_{i,k}\sqrt{P_{i,j}})^2\right)$, i.e., the k-th term in the argument of $\min\{\cdots\}$ in Equation (1). By the monotonicity of $C(\cdot)$, it is sufficient to consider $\sum_{j=1}^{k}(\sum_{i=0}^{j-1} h_{i,k}\sqrt{P_{i,j}})^2$. Now since the channel gains are

multiplicative, we have:

$$\sum_{j=1}^{k}(\sum_{i=0}^{j-1} h_{i,k}\sqrt{P_{i,j}})^2 = g_{0,k}\sum_{j=1}^{k}\left(\sum_{i=0}^{j-1}\frac{\sqrt{P_{i,j}}}{h_{0,i}}\right)^2$$

Thus our optimization problem becomes:

$$\begin{aligned}&\max \min_{k\in\{1,2,\cdots,N+1\}} g_{0,k}\sum_{j=1}^{k}\left(\sum_{i=0}^{j-1}\frac{\sqrt{P_{i,j}}}{h_{0,i}}\right)^2\\ &s.t. \quad \sum_{j=1}^{N+1}\gamma_j = P_T,\\ &and \quad \sum_{i=0}^{j-1} P_{i,j} = \gamma_j \,\forall\, j\in\{1,2,\cdots,N+1\} \end{aligned} \tag{11}$$

Let us fix $\gamma_1,\gamma_2,\cdots,\gamma_{N+1}$ such that their sum is equal to P_T. We observe that $P_{i,N+1}$ for $i\in\{0,1,\cdots,N\}$ appear in the objective function only once: for $k=N+1$ through the term $\left(\sum_{i=0}^{N}\frac{\sqrt{P_{i,N+1}}}{h_{0,i}}\right)^2$. Since we have fixed γ_{N+1}, we need to maximize this term over $P_{i,N+1}$, $i\in\{0,1,\cdots,N\}$. So we have the following optimization problem:

$$\begin{aligned}&\max \sum_{i=0}^{N}\frac{\sqrt{P_{i,N+1}}}{h_{0,i}}\\ &s.t. \quad \sum_{i=0}^{N} P_{i,N+1} = \gamma_{N+1}\end{aligned} \tag{12}$$

By Cauchy-Schwartz inequality, the optimal objective function in this optimization problem is upper bounded by $\sqrt{(\sum_{i=0}^{N}P_{i,N+1})(\sum_{i=0}^{N}\frac{1}{g_{0,i}})} = \sqrt{\gamma_{N+1}\sum_{i=0}^{N}\frac{1}{g_{0,i}}}$. The upper bound is achieved if there exists some $c>0$ such that $\frac{\sqrt{P_{i,N+1}}}{\frac{1}{h_{0,i}}} = c\ \forall i\in\{0,1,\cdots,N\}$. We have thus:

$$P_{i,N+1} = \frac{c^2}{g_{0,i}}\ \forall i\in\{0,1,\cdots,N\}$$

Using the fact that $\sum_{i=0}^{N}P_{i,N+1}=\gamma_{N+1}$, we obtain $c^2 = \frac{\gamma_{N+1}}{\sum_{l=0}^{N}\frac{1}{g_{0,l}}}$. As a consequence:

$$P_{i,N+1} = \frac{\frac{1}{g_{0,i}}}{\sum_{l=0}^{N}\frac{1}{g_{0,l}}}\gamma_{N+1} \tag{13}$$

Here we have used the fact that $h_{0,0}=1$. Now $\{P_{i,N}: i=0,1,\cdots,(N-1)\}$ appear only through the sum $\sum_{i=0}^{N-1}\frac{\sqrt{P_{i,N}}}{h_{0,i}}$, and it appears twice: for $k=N$ and $k=N+1$. We need to maximize this sum subject to the constraint $\sum_{i=0}^{N-1}P_{i,N} = \gamma_N$. This optimization can be solved in a similar way as before. Thus by repeatedly using this argument and solving optimization problems similar in nature to (12), we obtain:

$$P_{i,j} = \frac{\frac{1}{g_{0,i}}}{\sum_{l=0}^{j-1}\frac{1}{g_{0,l}}}\gamma_j\ \forall 0\le i<j\le(N+1) \tag{14}$$

Substituting for $P_{i,j}, 0\le i<j\le(N+1)$ in (11), we obtain the following optimization problem:

$$\begin{aligned}&\max \min_{k\in\{1,2,\cdots,N+1\}} g_{0,k}\sum_{j=1}^{k}\left(\gamma_j\sum_{i=0}^{j-1}\frac{1}{g_{0,i}}\right)\\ &s.t. \quad \sum_{j=1}^{N+1}\gamma_j = P_T\end{aligned} \tag{15}$$

Let us define $b_k := g_{0,k}$ and $a_j := \sum_{i=0}^{j-1}\frac{1}{g_{0,i}}$. Observe that b_k is decreasing and a_k is increasing with k. Let us define:

$$s_k(\gamma_1,\gamma_2,\cdots,\gamma_{N+1}) := b_k\sum_{j=1}^{k}a_j\gamma_j \tag{16}$$

With this notation, our optimization problem becomes:

$$\begin{aligned}&\max \min_{k\in\{1,2,\cdots,N+1\}} s_k(\gamma_1,\gamma_2,\cdots,\gamma_{N+1})\\ &s.t. \quad \sum_{j=1}^{N+1}\gamma_j = P_T\end{aligned} \tag{17}$$

Claim 1: Under optimal allocation of $\gamma_1,\gamma_2,\cdots,\gamma_{N+1}$ for the optimization problem (17), $s_1=s_2=\cdots=s_{N+1}$.

Proof: Problem (17) can be rewritten as:

$$\begin{aligned}&\max \zeta\\ &s.t. \quad \zeta \le b_k\sum_{j=1}^{k}a_j\gamma_j\ \forall\, k\in\{1,2,\cdots,N+1\},\\ &\sum_{j=1}^{N+1}\gamma_j \le P_T,\\ &\gamma_j\ge 0\ \forall\, j\in\{1,2,\cdots,N+1\}\end{aligned} \tag{18}$$

The dual of this linear program is given by:

$$\begin{aligned}&\min P_T\theta\\ &s.t. \quad \sum_{k=1}^{N+1}\mu_k = 1,\\ &a_l\sum_{k=l}^{N+1}b_k\mu_k + \nu_l = \theta\,\forall\, l\in\{1,2,\cdots,N+1\},\\ &\theta\ge 0,\\ &\mu_l\ge 0, \nu_l\ge 0\,\forall\, l\in\{1,2,\cdots,N+1\}\end{aligned} \tag{19}$$

Now let us consider a primal feasible solution $(\{\gamma_j^*\}_{1\le j\le N+1},\zeta^*)$ such that $b_k\sum_{j=1}^{k}a_j\gamma_j^* = \zeta^*$ for $k\in\{1,2,\cdots,N+1\}$, and $\sum_{j=1}^{N+1}\gamma_j^* = P_T$.

Also, let us consider a dual feasible solution $(\{\mu_j^*,\nu_j^*\}_{1\le j\le N+1},\theta^*)$ such that $\nu_l^* = 0$ for $l\in\{1,2,\cdots,N+1\}$, $\sum_{k=1}^{N+1}\mu_k^*=1$, $a_l\sum_{k=l}^{N+1}b_k\mu_k^* + \nu_l^* = \theta^*$ for $l\in\{1,2,\cdots,N+1\}$.

It is easy to check that $\zeta^* = P_T\theta^*$, which means that there is no duality gap for the chosen primal and dual variables. Since the primal is a linear program, the solution $(\gamma_1^*,\gamma_2^*,\cdots,\gamma_{N+1}^*,\zeta^*)$ is primal optimal. Thus we have established the claim, since the primal optimal solution satisfies it. ■

So let us obtain $\gamma_1, \gamma_2, \cdots, \gamma_{N+1}$ for which $s_1 = s_2 = \cdots = s_{N+1}$. Putting $s_k = s_{k-1}$, we obtain:

$$b_k \sum_{j=1}^{k} a_j \gamma_j = b_{k-1} \sum_{j=1}^{k-1} a_j \gamma_j \tag{20}$$

Writing $d_k := \frac{(b_{k-1} - b_k)}{b_k} \frac{1}{a_k}$ and using Equation (20):

$$\gamma_k = d_k \sum_{j=1}^{k-1} a_j \gamma_j \tag{21}$$

From this recursive equation we have:

$$\gamma_2 = d_2 a_1 \gamma_1 \tag{22}$$

and in general for $k \geq 3$,

$$\gamma_k = d_k a_1 \Pi_{j=2}^{k-1} (1 + a_j d_j) \gamma_1 \tag{23}$$

Using the fact that $\gamma_1 + \gamma_2 + \cdots + \gamma_{N+1} = P_T$, we obtain:

$$\gamma_1 = \frac{P_T}{1 + d_2 a_1 + \sum_{k=3}^{N+1} d_k a_1 \Pi_{j=2}^{k-1} (1 + a_j d_j)} \tag{24}$$

Thus if $s_1 = s_2 = \cdots = s_{N+1}$, there is a unique allocation $\gamma_1, \gamma_2, \cdots, \gamma_{N+1}$. So this must be the one maximizing R. Hence, optimum γ_1 is obtained by Equation (24). Then, substituting the values of $\{a_k : k = 0, 1, \cdots, N\}$ and $d_k : k = 1, 2, \cdots, N+1$ in Equations (23) and (24), we obtain the values of $\gamma_1, \gamma_2, \cdots, \gamma_{N+1}$ as shown in Theorem 4. Now under these optimal values of $\gamma_1, \gamma_2, \cdots, \gamma_{N+1}$, all terms in the argument of $\min\{\cdots\}$ in (1) are equal. So we can consider the first term alone. Thus we obtain the expression for R optimized over power allocation among all the nodes for fixed relay locations as : $R_{P_T}^{opt}(y_1, y_2, \cdots, y_N) = C\left(\frac{g_{0,1} P_{0,1}}{\sigma^2}\right) = C\left(\frac{g_{0,1}\gamma_1}{\sigma^2}\right)$. Substituting the expression for γ_1 from Equation (8), we obtain the achievable rate (9).

B. Proof of Theorem 7

When N relay nodes are uniformly placed along a line, we will have $y_k = \frac{kL}{N+1}$. Then our formula for achievable rate $R_{P_T}^{opt}(y_1, y_2, \cdots, y_N)$ for total power constraint becomes: $R_N = C(\frac{P_T}{\sigma^2} \frac{1}{f(N)})$ where $f(N) = a + \sum_{k=2}^{N+1} \frac{a^k - a^{k-1}}{1 + a + \cdots + a^{k-1}}$ with $a = e^{\frac{\rho L}{N+1}}$.
Since $a > 1$, we have :

$$\begin{aligned} f(N) &= a + \sum_{k=1}^{N} \frac{a^{k+1} - a^k}{1 + a + \cdots + a^k} \\ &= a + (a-1)^2 \sum_{k=1}^{N} \frac{a^k}{a^{k+1} - 1} \\ &\geq a + (a-1)^2 \sum_{k=1}^{N} \frac{a^k}{a^{k+1}} \\ &= a + (a-1)^2 \frac{N}{a} \end{aligned}$$

Thus we obtain:

$$\liminf_N f(N) \geq \lim_{N \to \infty} \left(e^{\frac{\rho L}{N+1}} + \frac{N}{e^{\frac{\rho L}{N+1}}} (e^{\frac{\rho L}{N+1}} - 1)^2 \right) = 1$$

On the other hand, noting that $a > 1$, we can write:

$$\begin{aligned} f(N) &= a + (a-1)^2 \sum_{k=1}^{N} \frac{a^k}{a^{k+1} - 1} \\ &\leq a + (a-1)^2 \sum_{k=1}^{N} \frac{a^{k+1}}{a^{k+1} - 1} \\ &= a + (a-1)^2 \sum_{k=1}^{N} \left(1 + \frac{1}{a^{k+1} - 1}\right) \\ &\leq a + (a-1)^2 \sum_{k=1}^{N} \left(1 + \frac{1}{a^{k+1} - a^k}\right) \\ &= a + N(a-1)^2 + (a-1) \sum_{k=1}^{N} a^{-k} \\ &= a + N(a-1)^2 + (a-1) a^{-1} \frac{1 - a^{-N}}{1 - a^{-1}} \end{aligned}$$

Hence,

$$\begin{aligned} \limsup_N f(N) &\leq \lim_{N \to \infty} \left(e^{\frac{\rho L}{N+1}} + N(e^{\frac{\rho L}{N+1}} - 1)^2 + (1 - e^{\frac{-\rho N L}{N+1}}) \right) \\ &= 2 - e^{-\rho L} \end{aligned}$$

Thus we have $\limsup_N R_N \leq C\left(\frac{P_T}{\sigma^2}\right)$ and $\liminf_N R_N \geq C\left(\frac{P_T/\sigma^2}{2 - e^{-\rho L}}\right)$. Hence the theorem is proved. □

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