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An Auction Framework for Spectrum Allocation with Interference Constraint in Cognitive Radio Networks

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***Abstract*—Extensive research in recent years has shown the benefits of *cognitive radio* technologies to improve the flexibility and efficiency of spectrum utilization. This new communication paradigm, however, requires a well-designed spectrum allocation mechanism. In this paper, we propose an auction framework for cognitive radio networks to allow unlicensed secondary users (SUs) to share the available spectrum of licensed primary users (PUs) fairly and efficiently, subject to the interference temperature constraint at each PU. To study the competition among SUs, we formulate a non-cooperative multiple-PU multiple-SU auction game and study the structure of the resulting equilibrium by solving a non-continuous two-dimensional optimization problem. A distributed algorithm is developped in which each SU updates its strategy based on local information to converge to the equilibrium. We then extend the proposed auction framework to the more challenging scenario with free spectrum bands. We develop an algorithm based on the no-regret learning to reach a correlated equilibrium of the auction game. The proposed algorithm, which can be implemented distributedly based on local observation, is especially suited in decentralized adaptive learning environments as cognitive radio networks. Finally, through numerical experiments, we demonstrate the effectiveness of the proposed auction framework in achieving high efficiency and fairness in spectrum allocation.**

I. Introduction

Cognitive radio [1] has emerged in recent years as a promising paradigm to enable more efficient and spectrum utilization. Apart from the conventional command and control model, three more flexible spectrum management models are presented in [2], namely, exclusive use (or operator sharing), commons and shared use of primary licensed spectrum. In the last model, unlicensed secondary users (SUs) are allowed to access the spectrum of licensed primary users (PUs) in an opportunistic way. In such a model, a well-designed spectrum allocation mechanism is crucial to achieve efficient spectrum usage and harmonious coexistence of PUs and SUs. On one hand, the radio resource allocation mechanism should ensure that the spectrum resource (unused by PUs) is allocated efficiently and fairly among SUs. On the other hand, the communication of PUs should not be disturbed by the SUs.

In this paper, we tackle the challenging research problem of designing efficient spectrum allocation mechanism for cognitive radio networks. We consider a generic network scenario in which multiple PUs and SUs coexist. To use the spectrum resource efficiently, the SUs share the available spectrum of the PUs under the condition that the *interference temperature* constraint [3] is always satisfied at each PU, i.e. the total received power of the SUs at each PU should be kept under some threshold in order to protect the PU's traffic. The considered scenario can represent various network scenarios, e.g. the PUs are the access points of a mesh network and the SUs are the mobile devices.

In our work, we develop an auction framework to allow SUs to share the available spectrum of PUs. Under the proposed auction framework, each PU acts as a resource provider by (1) announcing a price and a reserve bid (2) allocating the received power as a function of the bids submitted by SUs. Each SU acts as a customer by (1) submitting a two-dimensional bid indicating which PU to bid for resource and how much to bid (2) paying the chosen PU an amount of payment proportional to the allocated resource and the announced price. To study the competition among SUs, we formulate a non-cooperative auction game and study the structure of the resulting Nash equilibrium (NE) by solving a non-continuous two-dimensional optimization problem. A distributed algorithm is developped in which each SU updates its strategy based on local information to converge to the NE. Our analysis can serve as a decision and control framework for the SUs to exploit the underutilized spectrum resource.

We then extend the proposed auction framework to the more challenging scenario with free spectrum bands. In this context, a SU should strike a balance between accessing a free spectrum band with more interference if the competitors take the same strategy, and paying more for communication gains by staying with a licensed band. We show that the *ping-pong effect* may occur under the best-response update, i.e., a SU keeps switching between the free band and a licensed band. To eliminate the ping-pong effect, we develop an algorithm based on the no-regret learning [4] to reach a correlated equilibrium (CE) [5] of the auction game. The proposed algorithm, which can be implemented distributedly and requires only local observation, is especially suited in decentralized adaptive learning environments as cognitive radio networks.

Due to their perceived fairness and allocation efficiency [6], auctions are among the best-known market-based mechanisms to allocate spectrum [7], [8], [9], [10], [11], [12]. In most proposed auctions, the spectrum resource is treated as goods in traditional auctions studied by economists, i.e., one licensed band (or a collection of multiple bands) is awarded to one SU. However, spectrum auction differs from conventional

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auctions in that it has to address radio interference. Spectrum auction is essentially a problem of interference-constrained resource allocation. Only a few papers have discussed spectrum auctions under interference constraint, among which [11] and [12] studied conflict-free spectrum allocation with high spectrum efficiency. [10] developed an auction-based spectrum sharing framework to allow a single spectrum manager to share its spectrum with a group of users, subject to the interference temperature constraint at the measurement point, a requirement proposed by FCC in [3]. Based on the same model as [10], our work is among the relative few that investigate the interference-constraint radio resource allocation problem under the auction framework. Compared with previous work, we make the following key contributions:

- Existing auction mechanisms mainly focus on single-PU scenario with very limited analytical and numerical studies on multiple-PU case. Our work, however, conduct an in-depth analysis on the spectrum auction for multiple PUs to allocate their spectrum to multiple SUs efficiently and fairly. As a distinctive feature of the proposed auction framework, the SUs' strategy (bid) is two-dimensional and non-continuous, leading to a competition scenario with more complex interactions among players and requiring an original study of the resulting equilibrium.
- We investigate the spectrum auction with free spectrum bands and develop a distributed adaptive algorithm based on no-regret learning to converge to a CE of the auction game. To the best of our knowledge, our work is the first to adapt the auction framework to address the spectrum sharing problem in heterogeneous environments with both licensed and free bands.

The rest of this paper is structured as follows. Section II presents our system model and auction framework followed by the formulation of the non-cooperative auction game. Section III solves the auction game and analyzes the structural properties of the resulting NE. Section IV extends our auction framework to the more challenging scenario with free spectrum bands. Simulation results are presented in Section V. Section VI concludes the paper.

II. System Model and Spectrum Auctions

This section introduces the notation and the system model of our work, followed by the presentation of the proposed spectrum auction framework and the formulation of the auction game under the framework.

A. Cognitive radio network model

We consider a cognitive radio network consisting of a set of primary users referred to as PUs and a set of secondary transmitter-receiver pairs referred to as secondary users or SUs. We use $\mathcal{N} = \{1, 2, \cdots, N\}$ and $\mathcal{M} = \{1, 2, \cdots, M\}$ to denote the PU set and the SU set, respectively. We use S_i and D_i to denote the transmitter and the receiver of SU $i \in \mathcal{M}$. Each PU $n \in \mathcal{N}$ operates on a spectrum band n with bandwidth B_n that is non-overlapped with the spectrum bands of other PUs, i.e. $n_1 \bigcap n_2 = \Phi, \forall n_1, n_2 \in \mathcal{N}$.[1]

[1]The extension of our analysis to the more competitive scenario where the PUs' bands are overlapped with each other is left for future work.

SU i's valuation of the spectrum is defined by a utility function $U_i(\gamma_i)$, where γ_i is the received signal-to-interference-plus-noise ratio (SINR) at SU i's receiver D_i. $U_i(\gamma_i)$ characterizes the application payoff (e.g. satisfaction level) of SU i from SINR γ_i. We assume $U_i(\gamma_i)$ is continuously differentiable, strictly increasing and concave in γ_i with $U_i(0) = 0$. For each SU i, the received SINR using PU n's band is given by

$$\gamma_i = \frac{p_i h_{ii}}{n_0 B_n + \sum_{j \neq i} p_j h_{ji}}, \tag{1}$$

where p_i denotes SU i's transmission power, h_{ji} denotes the channel gain from SU j's transmitter S_j to SU i's receiver D_i, n_0 denotes the background noise power spectral density.

In the considered scenario, to ensure that the transmissions of PUs are not significantly degraded by the SUs, an interference temperature constraint is imposed such that the total received power of SUs at PU n must satisfy

$$\sum_{i=1}^{M} p_i g_{in} \leq P_n \quad \forall n \in \mathcal{N},$$

where g_{in} is the channel gain from S_i to PU n, P_n is the tolerable interference threshold at PU n.

B. Spectrum auction framework

We apply auction mechanisms to tackle the spectrum allocation problem. By definition, an auction is a decentralized market mechanism for allocating resources and can be formulated as a non-cooperative game, where players are bidders, strategies are bids, both allocations and payments are functions of bids. A well-known auction is the Vickrey-Clarke-Groves (VCG) auction [6], which is shown to have social optimal outcome. However, the VCG auction requires global information to perform centralized computations. To overcome this limitation, two one-dimensional share auction mechanisms, namely the SINR auction and the power auction are proposed in [10] to study the spectrum allocation problem in single-PU networks. In the following, we extend the work of [10] to the multiple-PU scenario by proposing the two-dimensional SINR and power auction, as shown in Algorithm 1.[2]

Algorithm 1 Two-dimensional spectrum auction algorithm

Price announcing: Each PU n announces a reserve bid β_n and a price $\pi_n > 0$.

Bidding: Based on β_n and π_n, each SU i submits a bid (a_i, b_i) where $a_i \in \mathcal{N}$ and $b_i \geq 0$.

Spectrum allocation: Each SU i is allocated a transmission power p_i from PU a_i as follows:

$$p_i = \frac{P_{a_i}}{g_{ia_i}} \frac{b_i}{\sum_{j \in \mathcal{M}, a_j = a_i} b_j + \beta_{a_i}}. \tag{2}$$

Payment collection: Each SU i pays PU a_i a payment $C_i = \pi_{a_i} \gamma_i g_{ia_i}$ in the SINR auction and $C_i = \pi_{a_i} p_i g_{ia_i}$ in the power auction.

[2]In our study, we assume that SUs are honest, and indeed make the payments. We do not consider the issue of *payment enforcement*, which may require a separate mechanism and is beyond the scope of the paper.

Under the above auction framework, the received SINR of SU i is

$$\gamma_i = \frac{P_{a_i}\frac{h_{ii}}{g_{ia_i}}b_i}{n_0 B_{a_i}\left(\sum_{j\in\mathcal{M},a_j=a_i} b_j + \beta_{a_i}\right) + \sum_{j\in\mathcal{M},a_j=a_i,j\neq i} P_{a_i}\frac{h_{ji}}{g_{ja_i}}b_j}. \quad (3)$$

In contrast to [10] where SUs are charged the same price per unit SINR, we apply the economic concept of *price discrimination* in the proposed SINR auction by imposing g_{ia_i} as a user-dependent pricing factor on SU i. The design rationale is that for two SUs choosing the same PU, the SU causing more interference at the PU should be charged more per unit SINR than the SU causing less interference. As we will show via numerical experiments, this feature is especially suited in multi-PU case by resulting a more balanced equilibrium. For the power auction, noticing that the received power of SU i at PU a_i is $p_i g_{ia_i}$, the auction scheme actually inplements a pricing policy under which a price π_n per unit received power is imposed by PU n to the SUs connecting to it.

C. Non-cooperative spectrum auction game formulation

Under the proposed auction framework, we model the interaction among SUs as a non-cooperative spectrum auction game, denoted as G_{NSA} and G_{NPA} for the SINR and power auction, respectively.[3] Let $s_i = (a_i, b_i)$ denote the strategy of SU i and s_{-i} denote the strategy of the SUs except i, given the price vector $\pi = (\pi_n, n \in \mathcal{N})$, each SU i chooses its strategy s_i to maximize his *surplus function* defined as follows:

$$S_i(s_i, s_{-i}) = U_i(\gamma_i(s_i, s_{-i})) - C_i(s_i, s_{-i}).$$

The resulting non-cooperative SINR (power) auction game can then be defined formally as:

$$G_{NSA}(G_{NPA}): \max_{s_i=(a_i,b_i),a_i\in\mathcal{N},b_i\geq 0} S_i(s_i, s_{-i}),\ \forall i \in \mathcal{M}.$$

The solution of the auction game is characterized by a Nash Equilibrium (NE), a strategy profile $\mathbf{s}^* = (s_i^*, s_{-i}^*)$ from which no player has incentive to deviate unilaterally [13], i.e.,

$$S_i(s_i^*, s_{-i}^*) \geq S_i(s_i, s_{-i}^*), \quad \forall i \in \mathcal{M}, \forall a_i \in \mathcal{N}, \forall b_i \geq 0.$$

As a distinguished feature from the single-PU auction, the auction framework proposed in our work is two-dimensional and involves both PU selection and bid adjustment, which leads to a competition scenario with more complex interactions among players. Consequently, characterizing structural properties of the auction game in our context requires an original study of the game equilibria that cannot draw on existing well-known results, as will be shown in later analysis.

III. Solving the Auction Game: NE Analysis

In this section, we solve the auction game by deriving the NE of the game and study the structure properties of the NE. To this end, we focus on the following optimization problem faced by each SU i in the spectrum auction game, given the price of PUs $\pi = \{\pi_n, n \in \mathcal{N}\}$ and strategies of others s_{-i}:

$$s_i^* = (a_i^*, b_i^*) = \operatorname*{argmax}_{s_i}\ S_i(s_i, s_{-i}), \quad (4)$$

which, according to the following lemma, can be written as

$$s_i^* = (a_i^*, b_i^*) = \operatorname*{argmax}_{a_i\in\mathcal{N}} \operatorname*{argmax}_{b_i\geq 0}\ S_i(s_i, s_{-i}).$$

Lemma 1. $\max_{(a_i,b_i)}\ S_i(s_i, s_{-i}) = \max_{a_i\in\mathcal{N}} \max_{b_i\geq 0}\ S_i(s_i, s_{-i})$.

Proof: On one hand, it follows from (4) that

$$S_i((a_i^*, b_i^*), s_{-i}) \geq \max_{a_i\in\mathcal{N}} \max_{b_i\geq 0}\ S_i((a_i, b_i), s_{-i}).^4$$

On the other hand, we have

$$\begin{aligned}\max_{a_i\in\mathcal{N}} \max_{b_i\geq 0}\ S_i((a_i, b_i), s_{-i}) &\geq \max_{b_i\geq 0}\ S_i((a_i^*, b_i), s_{-i}) \\ &\geq S_i((a_i^*, b_i^*), s_{-i}).\end{aligned}$$

Combining the above results completes our proof. ■

A. SINR auction

We start with the SINR auction game. Unlike the single-PU auction studied in [10], where each SU maximizes its surplus function over its bid only, the SU optimization problem in the multiple-PU case is a joint two-dimensional problem over the submitted bid and the PU to whom the SU bids for spectrum. To solve the SUs' optimization problem, a straightforward way to find (a_i^*, b_i^*) is to search over all possible PU settings and perform optimization over bid for every setting, which is computationally intensive and makes the resulting NE intractable. In our analysis, we overcome this technical difficulty by decomposing the two-dimensional optimization problem based on the structural properties of the surplus function, detailed in Lemma 2.

Lemma 2. *For each SU i, given π and s_{-i}, it holds that*

$$a_i^* = \operatorname*{argmax}_{n\in\mathcal{N}}\ S_i(\gamma_{in}^*) = \operatorname*{argmax}_{n\in\mathcal{N}}\ U_i(\gamma_{in}^*) - \pi_n g_{in}\gamma_{in}^*,$$

where $\gamma_{in}^* = \min\{U_i'^{-1}(\pi_n g_{in}), P_n h_{ii}/(n_0 B_n g_{in})\}, \forall n \in \mathcal{N}$.

Proof: Let γ_{in} denote the SINR of SU i when connecting to PU n, recall (3), we can show that:

1) γ_{in} is upper-bounded by $P_n h_{ii}/(n_0 B_n g_{in})$;
2) For $\gamma_{in} \leq P_n h_{ii}/(n_0 B_n g_{in})$, there is an one-to-one mapping between γ_{in} and b_i.

From Lemma 1, the optimization problem of SU i is thus equivalent to the following one:

$$\max_{n\in\mathcal{N}} \max_{\gamma_{in}\leq\frac{P_n h_{ii}}{n_0 B_n g_{in}}} S_i(n, \gamma_{in}).$$

Moreover, when choosing PU n, S_i can be written as a function of γ_{in} as

$$S_i(\gamma_{in}) = U_i(\gamma_{in}) - \pi_n g_{in}\gamma_{in},$$

whose derivative is

$$\frac{\partial S_i}{\partial \gamma_{in}} = U_i'(\gamma_{in}) - \pi_n g_{in}.$$

Following the concavity of U_i, U_i' is monotonously decreasing in γ_{in}. Hence S_i is a quasi-concave function of γ_{in}, thus has a unique global maximizer $\gamma_{in}^* =$

[3] In this work, we do not consider the PUs as players. A significant extension of our work presented in this paper is to model the spectrum auction as a *Stackelberg game*, in which the PUs are the leaders that choose their strategy (price) first, and the SUs are the followers that respond by choosing their strategies (bids) accordingly, knowing the leaders' strategies [13]. We leave this extension of exploring the Stackelberg game for future work.

[4] For the sake of simplicity, in case of non-ambiguity, we note $S_i((a_i^*, b_i^*), s_{-i})$ as a function of s_i, i.e. $S_i(s_i)$ or $S_i(a_i^*, b_i^*)$.

$\min\{U_i'^{-1}(\pi_n g_{in}), P_n h_{ii}/(n_0 B_n g_{in})\}$. The maximum of S_i under PU n is given by $S_i(\gamma_{in}^*)$. It then follows that

$$a_i^* = \underset{n\in\mathcal{N}}{\operatorname{argmax}}\ S_i(\gamma_{in}^*) = \underset{n\in\mathcal{N}}{\operatorname{argmax}}\ U_i(\gamma_{in}^*) - \pi_n g_{in}\gamma_{in}^*,$$

where $\gamma_{in}^* = \min\{U_i'^{-1}(\pi_n g_{in}), P_n h_{ii}/(n_0 B_n g_{in})\}$. ∎

Specifically, when π_n is significantly large, more precisely, $\pi_n g_{in} \geq U_i'(P_n h_{ii}/n_0 B_n g_{in}), \forall n \in \mathcal{N}, \forall i \in \mathcal{M}$, Lemma 2 can be simplified to Corollary 1.

Corollary 1. *If $\pi_n g_{in} \geq U_i'(P_n h_{ii}/n_0 B_n g_{in}), \forall n \in \mathcal{N}, \forall i \in \mathcal{M}$, it holds that $a_i^* = \operatorname{argmin}_{n\in\mathcal{N}} \pi_n g_{in}$.*

Proof: Recall that $U_i(\gamma_i)$ is concave in γ_i, $\pi_n g_{in} \geq U_i'(P_n h_{ii}/n_0 B_n g_{in})$ leads to $U_i'^{-1}(\pi_n g_{in}) \leq P_n h_{ii}/(n_0 B_n g_{in})$. It then follows from Lemma 2 that $\gamma_{in}^* = U_i'^{-1}(\pi_n g_{in})$ and

$$\begin{aligned} a_i^* &= \underset{n\in\mathcal{N}}{\operatorname{argmax}} S_i(U_i'^{-1}(\pi_n g_{in})) \\ &= \underset{n\in\mathcal{N}}{\operatorname{argmax}} U_i(U_i'^{-1}(\pi_n g_{in})) - \pi_n g_{in} U_i'^{-1}(\pi_n g_{in}). \end{aligned}$$

Let $x = \pi_n g_{in}$, regard $S_i = U_i(U_i'^{-1}(x)) - xU_i'^{-1}(x)$ as a function of x, after some mathematical operations, we have

$$\frac{\partial S_i}{\partial x} = -U_i'^{-1}(x),$$

which, following the concavity of U_i, is non-positive. $S_i(x)$ is thus non-increasing in x. Hence

$$a_i^* = \underset{n\in\mathcal{N}}{\operatorname{argmax}}\ S_i(U_i'^{-1}(\pi_n g_{in})) = \underset{n\in\mathcal{N}}{\operatorname{argmin}}\ \pi_n g_{in},$$

which concludes our proof. ∎

If we denote $\pi_n g_{in}$ as the effective price for SU i when choosing PU n, Corollary 1 states that SU i always chooses the PU with the minimum effective price.

As the key results of this subsection, we have demonstrated that in the SINR auction game, the choice of PU only depends on the effective price set by PUs. Consequently, the optimization problem of each SU i can be decomposed into two sub-problems, which can be performed sequentially:

1) i chooses PU a_i^* based on the effective price of PUs and stay with PU a_i^*;
2) i performs bid optimization by adjusting its bid submitted to PU a_i^*, which is degenerated into single-PU case.

The following theorem on the NE of the SINR auction game is then immediate whose proof follows straightforwardly from that of Theorem 1 and Proposition 6 in [10].

Theorem 1. *For the SINR auction with $\beta_n > 0, \forall n \in \mathcal{N}$, there exists a threshold price vector $\pi_{th}^s = \{\pi_{th,n}^s, n \in \mathcal{N}\}$ such that if the price vector $\pi > \pi_{th}^s$,[5] a NE exists to which the best response update converges. The NE is unique if a_i^* is singleton for every SU i. On the other hand, if there exists some $n_0 \in \mathcal{N}$ such that $\pi_{n_0} \leq \pi_{th,n_0}^s$, there is no NE.*

B. Power auction

In this subsection, we turn to the power auction game. As the payment function C_i in the power auction has a different structure to that in the SINR auction (i.e. C_i is a function of p_i instead of γ_i), the decomposition in the previous analysis on the SINR auction is no more applicable here. To characterize the equilibrium of the power auction game, we make the following approximation in the subsequent analysis:

$$\sum_{a_j=a_i, j\neq i} b_j \gg b_i, \forall i \in \mathcal{M}, \text{ or equivalently, } \sum_{s_j=a_i, j\neq i} b_j \sim \sum_{s_j=a_i} b_j. \quad (5)$$

The approximation (5) is accurate in large systems where the bid variation of any individual player has neglectable influence on the system state. More specifically, under (5), the impact of b_i on the interference at the receiver D_i, denoted as I_i, can be neglected, in other words, I_i can be regarded independent w.r.t. b_i. The utility function of SU i can then be written as:

$$S_i = U_i(\gamma_i) - \frac{\pi_{a_i} g_{ia_i} I_i}{h_{ii}}\gamma_i.$$

where $I_i = n_0 B_{a_i} + \sum_{j\in\mathcal{M}, j\neq i, a_j=a_i} p_j h_{ji}$.

To solve the power auction game, we transform the original game G_{NPA} into another game G'_{NPA} in which the strategy of SU i is (a_i, γ_i) instead of (a_i, b_i) in G_{NPA}. Under the approximation (5), γ_i can be regarded as a linear function of b_i. As I_i is independent w.r.t. b_i, any unilateral change in b_i can be transformed into related change in γ_i without any influence on γ_{-i}. Thus the original game G_{NPA} is equivalent to the transformed game G'_{NPA}, formally expressed as

$$G'_{NPA}: \quad \max_{s_i=(a_i,\gamma_i)} S_i(s_i, s_{-i}),\ i \in \mathcal{M}.$$

We now concentrate on the new game G'_{NPA}. Performing the same analysis as Lemma 1 and Corollary 1 by noticing that $I_i \geq n_0 B_{a_i}$, we have the following result that decouples the PU selection and the adjustment of γ_i in G'_{NPA}.

Lemma 3. *If $\pi_n g_{in} n_0 B_n / h_{ii} \geq U_i'(P_n h_{ii}/n_0 g_{in}), \forall n \in \mathcal{N}, \forall i \in \mathcal{M}$, it holds that $a_i^* = \operatorname{argmin}_{n\in\mathcal{N}} \pi_n g_{in} I_i / h_{ii}$.*

Compared with the SINR auction game where the effective price imposed by PU n to SU i is $\pi_n g_{in}$, in the power auction game, the corresponding effective price becomes $\pi_n g_{in} I_i / h_{ii}$. Lemma 3 states that SU i always chooses the PU with the minimum effective price. Armed with Lemma 3, we can then establish the existence of NE in G'_{NPA} under the condition that the prices set by PUs are sufficiently high.

Theorem 2. *Under the approximation* (5) *and the condition in Lemma 3, G'_{NPA} admits a NE.*

Proof: For any SU i, under the strategy of others $s_{-i} = (a_{-i}, \gamma_{-i})$, it follows from Lemma 3 that i chooses PU $a_i^* = \min_{n\in\mathcal{N}} \pi_n g_{in} I_i / h_{ii}$, i.e., for any $a_i' \neq a_i^*$, it holds that

$$\frac{\pi_{a_i^*} h_{ia_i^*} I_i(a_i^*)}{h_{ii}} \leq \frac{\pi_{a_i'} h_{ia_i'} I_i(a_i')}{h_{ii}}.$$

It then follows that for any $\gamma_i \geq 0$

$$\begin{aligned} S_i(a_i^*, \gamma_i) = U_i(\gamma_i) - \frac{\pi_{a_i^*} h_{ia_i^*} I_i(a_i^*)}{h_{ii}}\gamma_i \geq \\ U_i(\gamma_i) - \frac{\pi_{a_i'} h_{ia_i'} I_i(a_i')}{h_{ii}}\gamma_i = S_i(a_i', \gamma_i), \end{aligned}$$

which implies that given the opponents' strategy, choosing PU a_i^* is always the dominating strategy for any γ_i.

On the other hand, performing the same analysis as Lemma 1, we can show that in G'_{NPA},

$$\max_{(a_i,\gamma_i)} S_i(s_i, s_{-i}) = \max_{\gamma_i} \max_{a_i} S_i(s_i, s_{-i}).$$

The optimization problem for SU i thus becomes

$$\max_{(a_i,\gamma_i)} S_i(s_i, s_{-i}) = \max_{\gamma_i}\ S_i(a_i^*, \gamma_i),$$

[5] Throughout the paper, the inequality between two vectors is defined as the inequality in all components of the vectors.

in which the utility function of SU i is $S_i(a_i^*, \gamma_i)$, which is concave in γ_i. Furthermore, it follows from $I_i \geq n_0 B_n$ and $p_i \leq P_n/g_{ia_i}$ when SU i chooses PU n that $\gamma_i \leq \max_{n\in\mathcal{N}} h_{ii}P_n/(g_{ia_i}n_0B_n)$. Thus the strategy space $\gamma = (\gamma_i, i \in \mathcal{M})$ is a nonempty, convex, and compact set. It then follows from Theorem 1 in [14] that G'_{NPA} admits a NE. ■

Due to the complexity of the power auction game in which each SU has to solve a two-dimentional, non-continuous and non-decomposable optimization problem, we do not have a formal proof of the uniqueness of the NE and the convergence under the best response update. However, our experiment results show that the convergence is achieved in the vast majority of cases (cf. Section V-C).

C. The two-level game model

To get more insight on the structure of the auction game, we introduce and analyze in this subsection the following two-level game model: the lower level bidding game under fixed PU setting (Definition 1) and the higher level PU selection game (Definition 2).

Definition 1. *Given a fixed PU setting $\mathbf{a} = \{a_i, i \in \mathcal{M}\}$, the bidding game, denoted as $G^B_{NSA}(\mathbf{a})$ and $G^B_{NPA}(\mathbf{a})$ for the SINR and power auction respectively, is a tuple $(\mathcal{M}, \mathcal{A}, \{S_i, i \in \mathcal{M}\})$, where the SU set $\mathcal{M}$ is the player set, $\mathcal{A} = [0, +\infty)^M$ is the strategy set, $\{S_i\}$ is the utility function set with S_i being the surplus function. Each player (SU) i selects its strategy (bid) $b_i \geq 0$ to maximize its utility S_i.*

The above defined bidding game can be analyzed in the same way as the single-PU bidding game presented in [10] with the following result on the NE.

Lemma 4. *For the SINR auction (for the power auction under the approximation (5)) with $\beta_n > 0, \forall n \in \mathcal{N}$, there exists a threshold price vector $\pi^{sb}_{th}(\mathbf{a})$ ($\pi^{pb}_{th}(\mathbf{a})$) such that there exists a NE to which the best response update converges if the price vector $\pi > \pi^{sb}_{th}(\mathbf{a})$ ($\pi > \pi^{pb}_{th}(\mathbf{a})$), there is no NE otherwise.*

Proof: The proof for the SINR auction follows immediately from Theorem 1 and Proposition 6 in [10]. For the power auction, we show that under the condition in the lemma, the best response function has the same structure as that in the SINR auction in [10] whose convergence to NE is proven (Theorem 1 in [10]). To this end, recall that under (5), the utility function can be written as

$$S_i = U_i(\gamma_i) - \frac{\pi g_{i0} I_i}{h_{ii}}\gamma_i,$$

where I_i is independent of b_i. For each SU i, we can solve the best response $b_i = B(b_{-i})$ as follows:

$$\begin{cases} b_i = +\infty & \text{if } \pi \leq \frac{h_{ii}}{I_i g_{i0}} U_i'(\frac{P_{a_i} h_{ii}}{n_0 B_{a_i} g_{ia_i}}) \\ \pi = \frac{h_{ii} U_i'(\gamma_i)}{I_i g_{i0}} & \text{if } \frac{h_{ii}}{I_i g_{i0}} U_i'(\frac{P_{a_i} h_{ii}}{n_0 B_{a_i} g_{ia_i}}) < \pi < \frac{h_{ii}}{I_i g_{i0}} U_i'(0) \\ b_i = 0 & \text{if } \pi \geq \frac{h_{ii}}{I_i g_{i0}} U_i'(0) \end{cases} \quad (6)$$

Noticing the structural similarity between (6) and (22) in [10], we can establish the existence of NE and the convergence to the NE under the best response update (6). ■

Definition 2. *The PU selection game, denoted as G^{PU}_{NSA} and G^{PU}_{NPA} for the SINR and power auction respectively, is a tuple $(\mathcal{M}, \mathcal{A} = \{A_i\}, \{\widehat{S}_i, i \in \mathcal{M}\})$, where $\mathcal{M}$ is the player set, $A_i = \mathcal{N}$ is the strategy set of SU i, the utility function of SU i is defined as $\widehat{S}_i(a_i, a_{-i}) \triangleq S_i(\mathbf{a}, \mathbf{b}^*)$ where $\mathbf{b}^*(\mathbf{a}) = \{b_i^*(\mathbf{a}), i \in \mathcal{M}\}$ denotes the NE of the bidding game under the PU setting $\mathbf{a}$. Each player (SU) i selects its strategy (PU) $a_i \in \mathcal{N}$ to maximize its utility $\widehat{S}_i$.*

To analyze the PU selection game, we write the optimization problem of each SU i as

$$\max_{a_i} \ \widehat{S}_i(a_i, a_{-i}) = \max_{a_i} \ S_i(\mathbf{a}, \mathbf{b}^*(\mathbf{a})).$$

Noticing that in the bidding game under PU setting $\mathbf{a}$, it holds that $S_i(\mathbf{a}, \mathbf{b}^*(\mathbf{a})) = \max_{b_i} S_i(a_i, b_i)$, we thus have

$$\max_{a_i} \ \widehat{S}_i(a_i, a_{-i}) = \max_{a_i} \max_{b_i} \ S_i(a_i, b_i),$$

which, according to Lemma 1, is the same optimization problem as for the global auction game analyzed previously. Hence, we can map the NE of the PU selection game and the corresponding bidding game to the NE of the global auction game, as stated in the following theorem.

Theorem 3. *Any (pure) NE of the auction game can be mapped to a (pure) NE of the PU selection game $\mathbf{a}^*$ and the corresponding NE of the bid game $\mathbf{b}^*(\mathbf{a}^*)$ under the PU setting $\mathbf{a}^*$, i.e., any pure NE of the power auction game can be expressed as $\mathbf{s}^* = (a_i^*, b_i^*(\mathbf{a}^*), i \in \mathcal{M})$.*

By decomposing the global auction game into the PU selection game and the bidding game, we introduce a two-level architecture into the spectrum auction problem, in which the higher level PU selection game is a finite strategy game. This hierarchicalization can help us analyze the spectrum auction in more complex scenarios, as explored in the next section.

IV. Spectrum auction with free spectrum bands

Until now, we have analyzed the spectrum auction game in which the unlicensed SUs purchase spectrum ressource from licensed PUs. In this section, we extend our auction framework to the more challenging scenario with free spectrum bands. In such context, the SUs have the choice between accessing the licensed spectrum bands owned by PUs which is charged as a function of the enjoyed SINR or received power at PUs, and switching to the unlicensed spectrum bands which are free of charge but become more crowded when more SUs operate in these spectrum bands. Consequently, the SUs should strike a balance between accessing the free spectrum bands with probably more interference and paying for communication gains by staying with the licensed bands. In the subsequent study, we assume that there is one free spectrum band available for all SUs. The extension to multiple free band case is straightforward.

We start with the SINR auction. In the new scenario with a free band, we define the spectrum band set $\mathcal{N} = \{1, \cdots, N, N+1\}$ where band 1 to N are the licensed bands processed by PU 1 to N, band $N+1$ denotes the free band with bandwidth B_{N+1}. Compared with the previous analysis without free spectrum band, each SU i has an additional choice of switching to band $N+1$ and the corresponding utility is

$$S_i(N+1) = U_i(\gamma_i), \quad (7)$$

where γ_i is the SINR of SU i. It is obvious to see that all SUs operating at B_{N+1} transmits at its maximum power, denoted

as $p_j^{max}, j \in \mathcal{M}$, to maximize their utility. Hence

$$\gamma_i = \frac{p_i^{max} h_{ii}}{n_0 B_{N+1} + \sum_{j \neq i, a_j = N+1} p_j^{max} h_{ji}}.$$

From Corollary 1, each SU i faces the choice of accessing the licensed band with minimum effective price and the free band $N+1$. As in Definition 1 and 2, we can define the corresponding PU selection game and bidding game in the new context[6]. The PU selection game is a finite strategy game and hence has at least one pure or mixed NE. By performing the same analysis as that in Section III-C, we can establish a mapping between a NE of the auction game and a NE of the PU selection game in the new context.

We then address the problem of how to reach a NE of the PU selection game, which is also a NE of the global auction game. We first notice that the myopic best response update in the PU selection game is not guaranteed to converge to a NE. In fact, during the course of PU selection, the SUs may notice that the utility of accessing a licensed spectrum is higher than staying in the free spectrum, and thus switch to the licensed spectrum accordingly. Since the SUs do this simultaneously, the free spectrum becomes under-loaded and the SUs will switch back to the free spectrum in the next iteration. This phenomenon, in which a player keeps switching between two strategies, is known as ping-pong effect.

To eliminate the ping-pong effect, we develop an algorithm based on the no-regret learning to converge to a correlated equilibrium (CE) of the PU selection game, which is shown to be a CE of the global auction game, too. Before presenting the proposed algorithm, we first provide a brief introduction on CE and no-regret learning.

A. Overview of correlated equilibrium

The concept of CE was proposed by Nobel Prize winner, Robert J. Aumann [5], in 1974. It is more general than NE. The idea is that a strategy profile is chosen randomly according to a certain distribution. Given the recommended strategy, it is to the players' best interests to conform with this strategy. The distribution is called CE, formally defined as follows.

Definition 3. *Let $G = (\mathcal{N}, (\Sigma_i, i \in \mathcal{N}), (S_i, i \in \mathcal{N}))$ be a finite strategy game, where $\mathcal{N}$ is the player set, Σ_i is the strategy set of player i and S_i is the utility function of i, a probability distribution p is a correlated equilibrium of G if and only if $\forall i \in \mathcal{N}$, $r_i \in \Sigma_i$, it holds that*

$$\sum_{r_{-i} \in \Sigma_{-i}} p(r_i, r_{-i})[S_i(r_i', r_{-i}) - S_i(r_i, r_{-i})] \leq 0, \ \forall r_i' \in \Sigma_i,$$

or equivalently,

$$\sum_{r_{-i} \in \Sigma_{-i}} p(r_{-i}|r_i)[S_i(r_i', r_{-i}) - S_i(r_i, r_{-i})] \leq 0, \ \forall r_i' \in \Sigma_i.$$

The second formula means that when the recommendation to player i is to choose action r_i, then choosing action $r_i' \neq r_i$ cannot lead to a higher expected payoff to i.

The CE set is nonempty, closed and convex in every finite strategy game. Moreover, every NE is a CE and corresponds to the special case where $p(r_i, r_{-i})$ is a product of each individual player's probability for different actions, i.e., the play of the different players is independent.

[6]For the free band, there is no bidding game, or alternertively, we can define a dumb bidding game for the free band, at the NE of which each SU choosing the free band submits 0 as bid and the utility is given by (7)

B. Overview of no-regret learning

The no-regret learning algorithm [4] is also termed regret-matching algorithm. The stationary solution of the no-regret learning algorithm exhibits no regret and the probability of choosing a strategy is proportional to the "regret" for not having chosen other strategies. For any two strategies $r_i \neq r_i'$ at any time T, the regret of player i for not playing r_i' is

$$R_i^T(r_i, r_i') \triangleq \max(D_i^T(r_i, r_i'), 0), \tag{8}$$

where

$$D_i^T(r_i, r_i') \triangleq \frac{1}{T} \sum_{t \leq T} (S_i^t(r_i', r_{-i}) - S_i^t(r_i, r_{-i})). \tag{9}$$

$D_i^T(r_i, r_i')$ has the interpretation of average payoff that player i would have obtained, if it had played r_i' every time in the past instead of r_i. $R_i^T(r_i, r_i')$ is thus a measure of the average regret. The probability that player i chooses r_i is a linear function of the regret. For every period T, define the relative frequency of players' strategy $\mathbf{r}$ played till T periods of time as follow:

$$z_T(\mathbf{r}) \triangleq \frac{1}{T} N(T, \mathbf{r}),$$

where $N(T, \mathbf{r})$ denotes the number of periods before T that the players' strategy is $\mathbf{r}$. *As an important property, z_T is guaranteed to converge almost surely (with probability one) to a set of CE in no-regret learning algorithm.*

C. Proposed algorithm based on no-regret learning

In this subsection, we develop an algorithm (Algorithm 2) based on no-regret learning and prove its convergence to a CE of the SINR auction game.

Algorithm 2 No-regret learning algorithm: SINR auction

Initialization: For each SU i, let p denote a random number between 0 and 1 and $a_i^* = \min_{n \in \mathcal{N}} \pi_n g_{in}$ (if a_i is not a singleton, randomly choose one), set $p^0_{a_i = a_i^*} = p$ and $p^0_{a_i = N+1} = 1 - p_{a_i}$. Let T_0 be a sufficient iteration duration.

for $t = kT_0$, $k = 1, 2, 3, \cdots$ **do**

Select spectrum a_i with probability $p_i^t(a_i)$ and use best-response update to converge to the NE of the bidding game.

When the NE is achieved after sufficient time, update the average regret R_i^t.

Let a_i^t denote the spectrum which SU i selects for iteration t, let μ be a large constant, calculate p_i^{t+1} as:

$$\begin{cases} p_i^{t+1}(a_i) & = \frac{1}{\mu} R_i^t, \quad \forall a_i \in \mathcal{N}, \ a_i \neq a_i^t \\ p_i^{t+1}(a_i) & = 1 - \sum_{n \in \mathcal{N}, n \neq a_i^t} p_i^{t+1}(n), \quad a_i = a_i^t \end{cases}$$

end for

Theorem 4. *There exists a threshold price vector π^{th} such that if the price vector $\pi > \pi^{th}$, the proposed algorithm converges surely to a CE of the SINR auction game.*

Proof: It follows from the structure of the bidding game that a threshold price vector π^{th} exists such that if the

price vector $\pi > \pi^{th}$, the convergence to the NE of the bidding game is guaranteed under the given spectrum setting. It then follows from the convergence property of the no-regret learning that the proposed algorithm converges surely to a CE of the PU selection game, denoted as $\mathbf{p}$, i.e.,

$$\sum_{a_j\in\mathcal{N},j\in\mathcal{M},j\neq i} p(a_{-i}|a_i)[S_i((a_i',{b_i'}^*),(a_{-i},{b'_{-i}}^*))- S_i((a_i,b_i^*),(a_{-i},b_{-i}^*))]\leq 0,\quad \forall a_i'\in\mathcal{N},$$

where b_i^* and ${b_i'}^*$ is the strategy of SU i at the NE of the bidding game under the spectrum setting (a_i,a_{-i}) and (a_i',a_{-i}), respectively. It follows from the NE definition of the bidding game that

$$S_i((a_i',{b_i'}^*),(a_{-i},{b'_{-i}}^*))=\max_{\gamma_i} U_i(\gamma_i)-\pi_{a_i'}g_{ia_i'}\gamma_i.$$

On the other hand, we have

$$S_i((a_i',b_i'),(a_{-i},b_{-i}^*))\leq \max_{\gamma_i} U_i(\gamma_i)-\pi_{a_i'}g_{ia_i'}\gamma_i,\ \forall b_i'\geq 0.$$

Hence, it holds that

$$\sum_{a_j\in\mathcal{N},j\in\mathcal{M},j\neq i} p(a_{-i}|a_i)[S_i((a_i',b_i'),(a_{-i},b_{-i}^*))- S_i((a_i,b_i^*),(a_{-i},b_{-i}^*))]\leq 0,\quad \forall a_i'\in\mathcal{N},\ \forall b_i\geq 0,$$

indicating that $\mathbf{p}$ is also a CE of the SINR auction game. ■

As a desirable property, Algorithm 2 can be implemented distributedly such that each SU i only needs to know the price vector π, its own channel gain h_{ii} and that between S_i and each PU n g_{in}. The best response update of the bidding game can be implemented distributedly at each SU i based on the knowledge of h_{ii} and g_{in}, the measurement of n_0 and the SINR γ_i, as detailed in [10]. We then show that the average regret can be calculated at each SU without any other information. Noticing (9) and recall the utility function of the PU selection game in Definition 2, it suffices to show that at each iteration t, $\Gamma_i^t(a_i^t,a_{-i}^t)\triangleq\sum_{k\leq t}\widehat{S}_i(a_i^t,a_{-i}^k),\forall a_i^t\in\mathcal{N}$ can be calculated distributedly.

In fact, at each iteration k, $\widehat{S}_i$ can be calculated as

$$S_i^k=\begin{cases}U_i(\frac{h_{ii}p_i^{max}}{I_i^{N+1}}) & a_i^t=N+1\\ U_i(\gamma_{ia_i^t}^*)-\pi_{a_i^t}g_{ia_i^t}\gamma_{ia_i^t}^* & a_i^t\neq N+1\end{cases},$$

where $\gamma_{ia_i^t}^*={U_i'}^{-1}(\pi_{a_i^t}g_{ia_i^t})$, I_i^{N+1} is the interference experienced by SU i when choosing the free band, which can be measured locally. Γ_i^t can then be calculated by induction as

$$\Gamma_i^t=\begin{cases}U_i^t(a_i^t,a_{-i}^t) & t=1\\ \Gamma_i^{t-1}(a_i^t,a_{-i}^{t-1})+U_i^t(a_i^t,a_{-i}^t) & t>1\end{cases}.$$

Consequently, the average regret can then be calculated based on only local measurement, which leads to the entirely distributed implementation of the proposed algorithm.

For the power auction, a similar distributed algorithm based on no-regret learning can be derived with convergence to a CE.

V. Simulation Analysis

In this section, we conduct simulations to evaluate the performance of the proposed auction framework and demonstrate some intrincical properties of the proposed auction framework, especially the fairness and efficiency, which are not explicitely addressed in the analytical part of the paper. After presenting the simulation setting, we introduce a reference power allocation scheme, called NAIVE, to which our proposed auction mechanisms are compared. In the first set of simulations, we consider an illustrative scenario to compare the SINR, power auctions with the NAIVE scheme. In the second set of simulations, we focus on the power auction in realistic network configurations with and without free spectrum band.

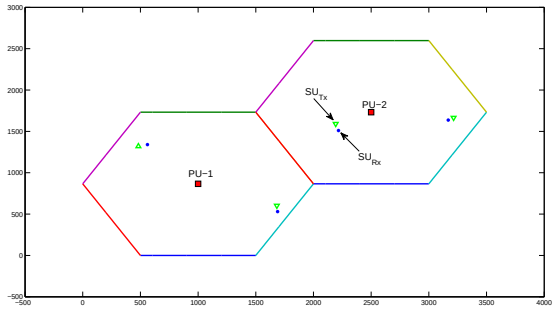


Fig. 1. Simulation setting

A. Simulation parameters and reference scheme

In our simulations, we consider a network of two PUs and multiple SUs (transmitter-receiver pairs). PUs can be seen as two access points or base stations covering two hexagonal cells, as shown in Figure 1. They can accept a certain amount of interference while allowing SUs to communicate during uplink PU transmissions.

In all simulations, we set $B_n=5$ MHz and $P_n=2n_0B_n$ $\forall n$. We adopt a typical urban path-loss model (C2 NLOS WINNER model [15] for WiMAX) with carrier frequency $f_c=3.5$ GHz and path-loss exponent $\alpha=3.5$. Shadowing effect is neglected.

In order to show the performance gain brought by our solutions, we introduce a reference power allocation scheme termed NAIVE. In NAIVE, SUs choose the furthest PU based on the knowledge of channel gains g_{in}[7]. Each PU n then allocates power $p_i=P_n/(M_ng_{in})$ to SU i choosing it, where M_n is the number of SU choosing PU n. In the scenario with a free band, the SUs in the NAIVE scheme switch to the free band with certain probability p_{free} (we analyze the cases $p_{free}=1/2$ and $p_{free}=1/3$). This simple scheme serves as the reference scheme for performance comparison.

B. Illustrative example: SINR and power auctions

We start with an illustrative example to compare the SINR, power auctions and the NAIVE scheme. We consider the fixed network configuration illustrated in Figure 1 with two PUs and four SUs with $\theta_i\in[1,20],\forall i$. There is no free band in this example. The prices $\pi_1=\pi_2$ are optimized by dichotomy[8].

We study the dynamics of the spectrum acution game under the best-response update. In the SINR auction, each SU chooses the PU with the minimum effective price (cf. Corollary 1) and then iteratively adjust its bid. Figure 2 (left) shows the convergence of allocated power to SUs. After about 40 iterations, convergence is reached. Compared with the

[7]The rationale of the choice is that choosing the furthest PU causes the least interference at the PU.

[8]Recall that the more competitive scenario where the PUs set their prices to maximize their revenue consists of a significant extension of the current work and is left for future studies.

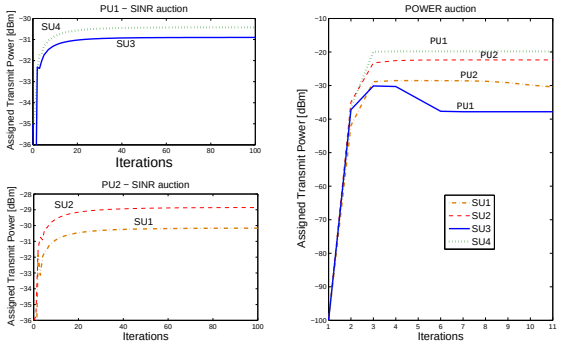


Fig. 2. Convergence of allocated power in the SINR auction (left) and power auction (right); final PU choice is shown for each SU with power auction.

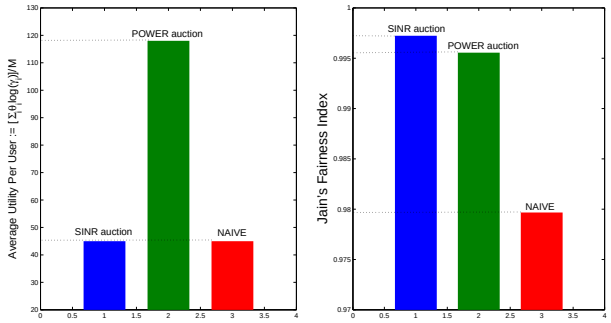


Fig. 3. Average utility per SU (left) and Jain's fairness index calculated based on U_i/θ_i (right) for SINR, power auctions and NAIVE

SINR auction where the choice of PU is done at the very first iteration and is not modified afterwards, in the power auction, the effective price is given by Lemma 3. As one part of the effective price, I_i changes from one iteration to another depending on the strategy of other SUs s_{-i}, thus the choice of PU may also vary from one iteration to another. However, as shown in Figure 2 (right), the final allocated power of each SU converges after about 10 iterations and the choice of PU is stablized. Compared with the SINR auction, the power auction converges in a faster but less smooth way.

In Figure 3, we focus on the efficiency and fairness of the considered schemes by studying the average utility per SU and the Jain's fairness index [16]. The Jain's index is computed based on the normalized utility U_i/θ_i. From the results, we observe that the SINR auction and the NAIVE scheme have almost the same average utility, but the SINR auction outperforms significantly the NAIVE scheme in terms of fairness. The power auction, on the other hand, has a very good performance in terms of both efficiency and fairness.

C. Realistic experiment: power auction

We now turn to more realistic scenarios. We focus on the power auction as it achieves the best performance in the above illustrative exemple. The power auction is also more natural and realistic in that SUs pay for the interference they create to PUs instead of the SINR they get as in the SINR auction. In our simulation, the transmitters of SUs are randomly located in each of the two cells. the receivers are randomly drawn in a disk with radius 100m whose center is the corresponding transmitter. We run Monte Carlo simulations with 1000 snapshots. At each snapshot, SU locations are randomly drawn with θ_i randomly drawn in $[1, 20]$.

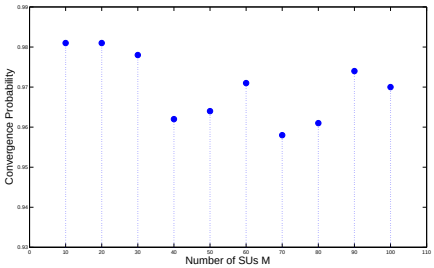


Fig. 4. Convergence probability of power auction

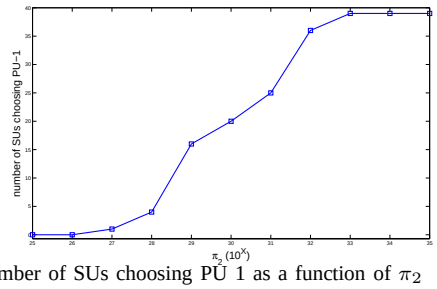


Fig. 5. Number of SUs choosing PU 1 as a function of π_2

1) Convergence: As explained in Section III-B, the best-response update is not guaranteed to converge. We thus study the convergence probability. We consider that the convergence is achieved if the best-response update in the power auction converges within 100 iterations, otherwise we consider that the auction does not converge. Figure 4 shows the probability of convergence as a function of the number of SU under this criterion: in the vast majority cases (more precisely, in more than 95% cases), convergence is achieved. In the subsequent simulations, in case of non-convergence, the results are based on the allocated power values after 100 iterations.

2) Load balancing: Figure 5 shows a scenario in which PU 1 fixes its price $\pi_1 = 10^{30}$ and PU 2 varies its price π_2 in the range $[10^{25}, 10^{35}]$. The total number of SUs M is set to 40. As shown in the figure, the number of SUs choosing PU 1 increases with π_2. The results demonstrate the benefit of the proposed power auction framework in load balancing by adjusting the prices of PUs. This feature is obviously not possible in NAIVE.

3) Efficiency and fairness: We now focus on two key performance metrics: efficiency and fairness. To this end, we compare the power auction and the NAIVE scheme in terms of average utility per SU and the Jain fairness index in two configurations. In the first configuration *M/2 system*, half of SUs are geographically located in cell 1 and the other half in cell 2. In the second configuration *M-2 system*, the number of SUs in cell 2 is constant ($M_2 = 2$), while the number of SU in cell 1 is variable in cell 1 ($M_1 = M - 2$). The two configurations represent two typical network scenarios, the balanced one with a homogeneous distribution of SUs and the unbalanced one with a heterogeneous distribution of SUs. As for the illustrative example, we set $\pi_1 = \pi_2$ and choose the price by dichotomy for the given number of SUs.

Figure 6 (left) shows that the average utility per SU is almost the same in the two configurations in the power auction (see the M/2=M-2 MultiPU Power curve in the figure) and is always higher than that in the NAIVE scheme. Figure 7 shows that the Jain fairness index (calculated in the same way as in the illustrative example) of power auction is always above that

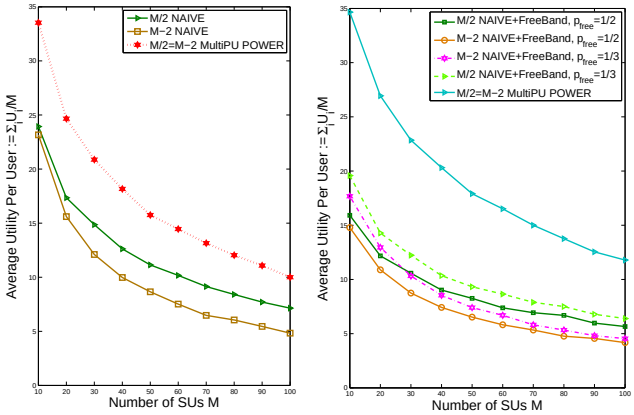


Fig. 6. Utility comparison in balanced (*M/2*) and unbalanced (*M-2*) scenarios between power auction and NAIVE with (left) and without (right) free band

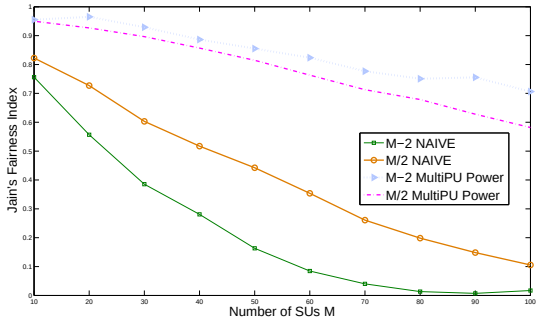


Fig. 7. Fairness comparison in balanced (*M/2*) and unbalanced (*M-2*) scenario between power auction and NAIVE

of NAIVE. In particular, in the unbalanced scenario, the power auction outperforms significantly the NAIVE scheme.

4) Power auction with a free band: We now study the power auction and the proposed no-regret learning algorithm (Section IV-C) by introducing a free band of 5 MHz. $p_i^{max} = 20$ dBm, $\forall i \in \mathcal{M}$. In the simulation, SUs in the NAIVE scheme choose the free band with probability $p_{free} = 1/2$ or $p_{free} = 1/3$ and emit at the maximum power p_i^{max}. The power allocation of SUs staying in licensed bands follows the same way as in the scenario without free band.

Figure 6 (right) shows the average utility of the power auction and NAIVE. As can be observed, compared with the scenario without free band, the average utility in NAIVE is slightly degraded even a new band is introduced. In contrast, the no-regret learning algorithm results a higher utility when the free band is added. Consequently, the utility gap between the power auction and NAIVE is more significant in the scenario with free band. Furthermore, we observe the convergence of the no-regret learning algorithm. Figure 8 shows the evolution of the number of SUs choosing PU1, PU2 and the free band for $M = 50$. The results demonstrate the benefit of the proposed no-regret learning algorithm to converge to an equilibrium with reasonable network efficiency in a distributed fashion.

VI. Conclusion

In this paper, we proposed an auction framework for cognitive radio networks to allow unlicensed SUs to share the available spectrum of licensed PUs, subject to the interference temperature constraint at each PU. We provided an in-depth analysis on the resulting multiple-PU multiple-SU non-cooperative auction game. We then extended the proposed auction framework to the more challenging scenario with free spectrum bands by developing an algorithm based on no-regret learning to reach a CE of the auction game. The proposed algorithm, which can be implemented distributedly based on local observation, is especially suited in decentralized adaptive learning environments as cognitive radio networks. The simulation results demonstrate the effectiveness of the proposed auction framework in achieving high efficiency and fairness in spectrum allocation.

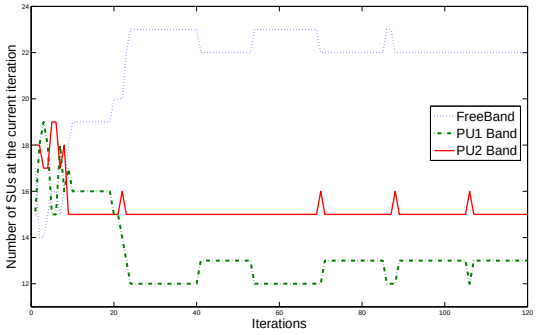


Fig. 8. Evolution of number of SUs choosing PU1, PU2 and the free band

As stated in the paper, a significant extension of our work is to study the more competitive Stackelberg game in which PUs choose their prices to maximize their revenue. Studying the efficiency of the spectrum auction in that scenario is the subjet of our on-going work.

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